The Pollution Premium

Po-Hsuan Hsu, Kai Li, and Chi-Yang Tsou

December 1, 2018

Abstract

This paper studies the implications of environmental pollution on the cross-section of stock returns. A long-short portfolio constructed from firms with high versus low toxic emission intensity within industry generates an average return of 5.52% per annum. To explain this pollution premium, we develop a general equilibrium asset pricing model in which firms’ cash flows face the uncertainty of policy regime shifts with respect to the environmental regulations. High emission (“dirty”) firms are more exposed to policy regime shift risks, and are therefore expected to earn higher average returns than those of low emission (“clean”) firms.

JEL Codes: E2, E3, E4, G1, Q5

Keywords: Toxic emissions, Regime shift risk, Uncertainty, Environmental regulation, Cross-section of stock returns

*Po-Hsuan Hsu (pohsuan.hsu@gmail.com), Faculty of Business and Economics, University of Hong Kong and College of Technology Management, National Tsing Hua University; Kai Li (kaili@ust.hk), School of Business and Management, Hong Kong University of Science and Technology; and Chi-Yang Tsou (bentsou@ust.hk), School of Business and Management, Hong Kong University of Science and Technology. We are grateful for helpful comments from Hengjie Ai, Ivan Alfaro, Frederico Belo, Patrick Bolton, Gautam Gowrisankaran, Thomas Hellmann, Mingyi Hung, Chuan Yang Hwang, Alan Kwan, Tse-Chun Lin, Evgeny Lyandres, Christopher Malloy, Kalina Manova, Roni Michaely, Ivan Png, Vesa Pursiainen, Tatsuro Senga, Lei Zhang, Lu Zhang, and seminar and conference participants at the NCTU International Finance Conference, the Taiwan Economics Research Conference, and the TFA Asset Pricing Symposium. We are responsible for any remaining errors or omissions.
1 Introduction

Firms that produce goods and products to satisfy consumer needs necessarily produce pollution as part of the industrial production process. Without the presence of environmental regulations, firms that maximize shareholder values have little incentive to reduce their environmental impacts. Presently, laws and regulations require firms to pay attention to environmental issues and internalize the social costs of the pollution they generate. That said, this emission regulation regime is significantly changing over time. For example, the regulation of EU’s Emissions Trading Scheme (ETS) was initially free but changed to auctions for carbon emission quotas.\(^1\) In this paper, we study the asset pricing implications of policy regime shift risks of such environmental regulation, especially through the lens of the cross-section of stock returns. We first establish a general equilibrium asset pricing model in which high emission firms’ profitability and, therefore, stock prices are more exposed to policy regime shift risks; as a result, our model rationalizes a pollution-return relation that we call the pollution premium.

We first develop a general equilibrium asset pricing model in which firms’ cash flows face the uncertainty of regime shifts with respect to environmental regulation. In our model, government (i.e., social planner) learns about the welfare costs of toxic emissions under a weak regulation regime in a Bayesian fashion by observing signals, and then actively makes an optimal decision between a strong or weak emission regulation regime. Adopting a strong regulation regime will lower emissions but negatively impact on all firms’ profitability, leading to a stronger negative impact on firms with high emissions. The government maximizes social welfare based on such a trade-off, as a social planner would do. In particular, we find that government, if acting optimally, replaces a weak regulation regime with a strong one if the environmental cost is perceived to be sufficiently high (i.e., the posterior mean of the pollution cost is above a given endogenous threshold). On the one hand, this shift to a strong regime is assumed to negatively affect the economy-wide average profitability, and therefore cause a upward spike in the stochastic discount factor; on the other hand, since high emission firms’ profitability is affected more than that of low emission firms, the former display a larger decline in stock prices upon a regime shift and are more negatively exposed to a regulation regime shift risk, which results in higher average excess returns ex-ante. This theoretical prediction is consistent with our empirical findings.

Our model is largely, but not solely, based upon that of Pástor and Veronesi (2012, 2013): we focus on the cross-sectional variation of expected stock returns, while they focus on the

---

\(^1\)The ETS is undoubtedly the most important environmental regulation, and ETS features different stages for enforcement. This ETS regulation begins with phase I and free carbon quotas for firms, and then ends with Phase III in which firms must access carbon emission quotas via auctions from 2013 onwards.

To study the empirical relation between toxic emissions and expected stock returns at the firm level, we construct a measure of emission intensity by using data from Environmental Protection Agency (EPA) and data from Compustat. We measure a firm’s toxic emissions by summing chemical emissions across all its plants that are listed in the EPA database. We then assign firms to different portfolios based on their ratios of chemical emissions over book equity relative to their industry peers, given that chemical emissions generally vary across industries. Such portfolio sorting shows that firms producing more pollution are associated with higher subsequent stock returns, and that the high-minus-low portfolio strategy based on simple emissions (toxicity-adjusted emissions) yields statistically significant average returns of 5.52% (5.87%) per annum. We also find significant alphas such that the high-minus-low portfolio is literally unaffected by known return factors for other systematic risks. These findings suggest a pollution-related risk premium driven by heterogeneous exposure to unspecified risks.

To assess whether the cross-sectional return predictive power of emission intensity is robust to a wider set of controls, we perform Fama and MacBeth (1973) regressions that control for industry effects and other known predictors, including size, book-to-market ratio, profitability, book leverage, R&D intensity, organization capital, asset growth, and investment intensity. We find that simple emissions and toxicity-adjusted emissions predict stock returns with strong statistical significance. In addition, one-standard-deviation increase in firm-level emission intensity increases future stock returns from 6.8% to 9.9% per year. Overall, we find that the emission-return relation remains economically and statistically significant, irrespective of the control variables that we consider.

Additional empirical analyses provide the following supportive evidence for our model assumptions and predictions. First, firm-level emissions negatively and significantly predict future profitability. Second, when a policy regime is more likely to shift (measured by a higher number of firms reporting emissions, higher temperatures, and more rainfalls), firms with higher emissions experience additional declines in future profits. Third, we verify the channel for the reduced profitability by showing that high emission firms are more likely involved in future litigations related to environmental issues. Last and most importantly, we show that high emission firms’ market values significantly decrease as the policy regimes shift.

In summary, our work identifies a new source of risk for investors: a regime shift risk of emission regulation policy that impacts higher emission firms greater than low emission
firms. With perspective to investment, firms with heavy pollution pose a risk since their profitability and stock prices are more negatively affected upon a regime shift from weak to strong emission regulation policy, as investors require higher expected stock returns to compensate for perceived risks. Hence, our pollution proxies, simple emissions and toxicity-adjusted emissions, carry risk characteristics distinct from characteristics documented in the literature.

This paper builds upon a growing literature stream that investigates between policy implications and environmental pollution. Most of these papers focus a great deal on the economic consequence of global warming and climate change. Acemoglu (2002) shows that two major forces bias the technological change: price effect and market size effect. Acemoglu, Aghion, Bursztyn, and Hemous (2012) suggest policy interventions to direct innovation from dirty technologies to clean ones, if two types of technologies are substitutable. If the dirty technology is more advanced, Acemoglu, Akcigit, Hanley, and Kerr (2016) show that interventions, including taxes and subsidies, promote transitions to clean technology. In their study of the automobile industry, Aghion, Dechezleprêtre, Hemous, Martin, and van Reenen (2016) find that cost-saving motivations encourage firms to develop clean technologies. These collective studies are quite different from ours, as none of them analyze the asset pricing implications. Unlike studies that consider carbon emissions, Currie, Davis, Greenstone, and Walker (2015) investigate the impact of toxic emissions on housing value and infant health; in contrast, we use toxic emissions data to study the implications of firms’ exposure to regulation regime shifts with respect to asset prices and returns.

Our work is connected to the literature that explores the asset pricing implications of social responsibility and climate change. Hong and Kacperczyk (2009) find that “sin” industries (i.e., alcohol, tobacco, and gaming) outperform non-“sin” industries since social norm dissuade institutional investors from investing in “sin” industries, which causes funding constraints for the “sin” industries. In their study of climate change’s impact on financial markets, Hong, Li, and Xu (2016) find evidence that food firms of drought-stricken countries underperform those of countries that do not experience droughts. Also, from the perspective of investment strategy, Andersson, Bolton, and Samama (2016) propose a hedging strategy against climate risks. Chava (2014), meanwhile, studies the impact of social responsibility on a firm’s cost of capital, and shows that firms with environmental concerns incur high equity and debt financing costs. On the other hand, Bansal and Ochoa (2011) and Bansal, Kiku, and Ochoa (2016) use climate change risks to embody long-run risks in dividends and consumption dynamics, so they may examine the implications of asset prices and social welfare. We differ from these papers in that our work concentrates on firms’ toxic emissions, studies cross-sectional asset pricing implications, and, more importantly, proposes a general
equilibrium model to explain that firms with high toxic emissions face more risk exposure with respect to regulation regime shifts.

Our paper is also related to asset pricing implications with macroeconomic uncertainty, a topic for which Pástor and Veronesi (2012, 2013) provide a comprehensive literature review. Brogaard and Detzel (2015) study asset pricing implications of the economic policy uncertainty index constructed by Baker, Bloom, and Davis (2016). Similar findings from Bali, Brown, and Tang (2017) suggest that uncertainty is priced in the cross-section by using the alternative measure of the economic uncertainty index proposed by Jurado, Ludvigson, and Ng (2015). From a theoretical perspective, Pástor and Veronesi (2012, 2013) show the impact of government policy uncertainty on asset prices with Bayesian learning. In addition, Liu, Shu, and Wei (2017) find direct evidence that stock prices of highly politically sensitive firms respond more to political uncertainty than those of firms with lower levels of political sensitivity. In contrast to these papers, our paper explores the financial effect of regulation uncertainty on toxic emissions.

Moreover, our paper contributes to the literature that relates consumption or productivity risk to aggregate- and firm-level risk premium by providing novel theoretical and empirical analyses on the role of pollution.

The rest of our paper is organized as follows. Section 2 presents the model. Section 3 introduces data construction and summary statistics. In Section 4, we present our empirical analysis. We conclude this paper with Section 5. The Appendix contains additional empirical evidence as well as our model solution.

---

2 There is a large literature on theories of macroeconomic uncertainty, but we do not attempt to summarize it here.

3 A large number of theoretical and empirical papers exist in the literature that relates consumption or productivity risk to the equity risk premium. Ait-Sahalia, Parker, and Yogo (2004) and Lochstoer (2009) show that luxury consumption can explain the equity premium. Yogo (2006) separates durable consumption from non-durable consumption to study time-series asset pricing implications, while Gomes, Kogan, and Yogo (2009) further show that durable good producers are riskier than non-durable good producers since the demand for durable goods is more pro-cyclical. Moreover, Savov (2011) uses garbage release data to capture volatile consumption, and Da, Yang, and Yun (2015) use electricity data to proxy for missing homemade goods. In addition, Kroencke (2016) suggests the unfiltered consumption to explain for why garbage data outperforms NIPA consumption data in matching the equity premium. These papers seek alternative proxies for smooth consumption and attain successful calibrations by generating sizable equity premium with reasonable risk aversion, and these papers even claim to explain cross-sectional stock returns (e.g., Fama-French 25 portfolios), as extensions. The literature also explores the asset pricing implications of production risk referred to as production-based asset pricing, which builds a bridge between investment and stock returns. Zhang (2005) provides an investment-based explanation for the value premium. Eisfeldt and Papanikolaou (2013) develop a model of organizational capital and expected returns. Kogan and Papanikolaou (2013, 2014) study the relation between investment-specific technology shocks and stock returns, so they may propose a fundamental explanation for the value premium. van Bingbergen (2016) documents the cross-sectional return spread by sorting on producer prices. Finally, Loualiche (2016) studies the cross-sectional difference in exposure to the globalization risk premium, and exemplifies such risk as an extension of the displacement risk proposed by Garleanu, Kogan, and Panageas (2012).
2 A General Equilibrium Model

In this section, we build a general equilibrium asset pricing model that features risk related to environmental policy regime shifts to explain the role of pollution with respect to stock prices and expected returns. Our specification of policy regime shifts is similar to that of Pástor and Veronesi (2012, 2013).

2.1 The Model Economy

We consider an economy with a finite horizon \([0, T]\) and a continuum of firms \(i \in [0, 1]\). Let \(B_i^t\) denote firm \(i\)'s capital at time \(t\). Debt financing is not taken into account, and firms in our economy entirely rely on equity financing. Therefore, \(B_i^t\) can also be regarded as the book value of equity. At time 0, all firms are endowed with the same amount of capital, which we normalize to \(B_0^i = 1\). Firm \(i\) invests its capital in a linear production technology with a stochastic rate of return denoted by \(d\Pi_i^t\). All profits are reinvested, so that firm \(i\)'s capital dynamics denote \(dB_i^t = B_i^t d\Pi_i^t\). Given that \(d\Pi_i^t\) equals profits over book equity, we refer to it as the profitability of firm \(i\). For all \(t \in [0, T]\), profitability then follows the process

\[
d\Pi_i^t = (\mu + \xi g) dt + \sigma dZ_t + \sigma_I dZ_i^t,
\]

in which \((\mu, g, \sigma, \sigma_I)\) are observable and constant parameters, \(Z_t\) is a Brownian motion, and \(Z_i^t\) is an independent Brownian motion that is specific to firm \(i\). The parameter \(g\) denotes the impact of different policy regime shifts (i.e., weak or strong environmental regulation) on the mean of the profitability process among firms. When \(g = 0\), the environmental policy regime is “neutral” with a zero impact on firm \(i\)'s profitability. The impact of environmental policy regime shifts, \(g\), is constant while the same regime is in effect. At time \(\tau\) (i.e., \(0 < \tau < T\)), the government makes an irreversible decision on whether or not to change its environmental policy from a weak to strong regulation. As a result, \(g\) is a simple step function of time:

\[
g = \begin{cases} 
g^W & \text{for } t \leq \tau \\
g^W & \text{for } t > \tau \text{ if there is no policy regime shift} \\
g^S & \text{for } t > \tau \text{ if there is a policy regime shift}, \end{cases}
\]

in which \(g^W\) denotes the impact of environmental policy under the weak regulation at the beginning. An environmental policy change replaces the weak regulation, \(W\), by the strong regulation, \(S\), and hence introduces a permanent drop in average profitability across firms. Such a policy decision is immediately effective when a regime shift occurs at time \(\tau\). We
assume that \( g^W > 0 \) and \( g^S < 0 \). This key assumption captures the idea that different environmental policies generate strong yet opposite impacts on firms’ profitability. To illustrate this point, the parameter \( \xi^i \) governs firm \( i \)'s exposure to environmental policy regime shifts. We assume that \( \xi^i \)’s are determined by emission levels that \( \xi^i \)’s are drawn from a uniform distribution on the interval \([\xi_{\min}, \xi_{\max}]\) at time 0 and then remain unchanged. Without a loss of generality, we then normalize the cross-sectional distribution of \( \xi^i \)’s with a mean equal to 1. We assume that there are two firms: a high emission firm (i.e. \( \xi^H > 0 \)) and a low emission firm (i.e. \( \xi^L \) such that \( \xi^L < \xi^H \)). Owing to a lower abatement cost under the weak regime, the high emission firm’s average profitability is higher than that of the low emission firm by the magnitude of \( g^W \)(i.e., \( \xi^H - \xi^L > 0 \)). In stark contrast, because of \( g^S < 0 \) under the strong regime, the high emission firm’s average profitability drops more than that of the low emission firm. As \( \xi^i \) is drawn from a uniform distribution with the mean normalized to 1, environmental policy regime shifts trigger an adverse effect on the average profitability in the economy. In particular, the high emission firm’s profitability is more subject to regime shifts. On the other hand, since \( \xi^i \) could be negative for the low emission firm, switching from a weak to a strong regime positively impacts the low emission firm’s average profitability when \( \xi^i g^S \) is positive. Taken together, the cross-sectional dispersion in firms’ exposures to regime shifts, \( \xi^i \)’s, serves an important driving force to determine different risk premia in equilibriums.

The firms are owned by a continuum of identical households who maximize expected utility derived from terminal wealth.\(^5\) For all \( j \in [0, 1] \), investor \( j \)'s utility function is given by

\[
U(W^j_T) = \left(\frac{W^j_T}{1 - \gamma}\right)^{1-\gamma},
\]

for which \( W^j_T \) is investor \( j \)'s wealth at time \( T \), and \( \gamma > 1 \) is the coefficient of relative risk aversion. At time 0, all investors are equally endowed with shares of firm stocks. Stocks pay dividends at time \( T \).\(^6\) Households observe whether regime shifts take place at time \( \tau \).

When making its policy decision at time \( \tau \), the government maximizes the same objective function as households, except that it also faces a environmental cost \( \Phi(C) \) associated with regime shifts. The government commits to a change in environmental regulation only if households’ expected utilities under the strong regulation is higher than under weak regulation.

---

\(^5\)This setting is consistent in our empirical design of scaling emissions by book equity.

\(^6\)No dividends are paid before time \( T \) because households’ preferences do not involve intermediate consumption. Firms in our model reinvest all of their earnings, as mentioned earlier.
Specifically, the government solves the optimization problem

$$\max_{\tau > t} \left\{ E_{\tau} \left[ \frac{\Phi(C)W_{T}^{1-\gamma}}{1 - \gamma} \right| W, E_{\tau} \left[ \frac{W_{T}^{1-\gamma}}{1 - \gamma} \right| S \right] \right\},$$

in which $W_{T} = B_{T} = \int_{0}^{1} B_{i} \, di$ is the final value of aggregate capital, $C$ is the environmental cost (i.e., collateral damage) if the weak regulation is retained, and $\Phi(C) = 1 + C$ is the corresponding cost function. We refer to $\Phi(C) = 1 + C > 1$ as a cost for households because, given $\gamma > 1$ and the lognormal distribution assumption, a higher value of $\Phi(C)$ translates into lower utility since $W_{T}^{1-\gamma}/(1 - \gamma) < 0$. The value of $C$ is randomly drawn at time $\tau$ from a lognormal distribution centered at $C = 1$.

$$c \equiv \log(C) \sim \text{Normal} \left( -\frac{1}{2} \sigma_{c}^{2}, \sigma_{c}^{2} \right),$$

in which $c$ is independent of the Brownian motions in equation (1). As soon as the value of $c$ is revealed to all agents at time $\tau$, the government uses this value to make its environmental policy decisions. We refer to $\sigma_{c}$ as regime shifts uncertainty. Regime shifts uncertainty introduces an element of surprise into firms’ valuations.

### 2.2 Learning about Environmental Costs

At time $t < \tau$, the government starts to learn about $c$ by observing unbiased signals. We model these signals as the true value of signal plus noise, which takes the following form in continuous time:

$$ds_{t} = cd_{t} + \eta dZ_{t}^{c}.$$  

The signal $ds_{t}$ is independent to other shocks in the economy. We refer to these shocks as environmental cost signals, and note that they capture the steady flow of news related to environmental issues that are of deep concern to both the public media and regulation authorities. Combining the signals in equation (6) with the prior distribution in equation (5), we obtain the posterior distribution of $c$ at any time $t < \tau$:

$$c \sim \text{Normal}(\hat{c}_{t}, \hat{\sigma}_{c,t}^{2}),$$

in which the posterior mean and variance evolve as

$$d\hat{c}_{t} = \hat{\sigma}_{c,t}^{2} \eta^{-1} d\hat{Z}_{t}^{c},$$

in which the posterior mean and variance evolve as
\[ \hat{\sigma}_{c,t}^2 = \frac{1}{\sigma_c^2 + \tau^2}. \]  

Equation (8) shows that the government’s beliefs about \( c \) are driven by the Brownian motion shocks \( d\hat{Z}_t \), which reflect the differences between the cost signals \( ds_t \) and their expectations \( (d\hat{Z}_t = \eta^{-1}(ds_t - E_t[ds_t])) \). Since the cost signals are independent of all fundamental shocks in the economy (i.e., \( dZ_t \) and \( dZ_i \)), the innovations \( d\hat{Z}_t \) represent signal shocks to the true value of environmental costs. These shocks shape the government’s beliefs about which environmental policy is likely to be adopted in the future, above and beyond the effect of fundamental economic shocks. Such a shift in belief alters the government’s decision-making, which therefore changes the probability of policy regime shifts. Accordingly, we now refer such signal shocks as policy regime shift shocks.

### 2.3 Optimal Regulation Regime Changes

After a period of learning about \( c \), the government decides whether to implement policy regime shifts at time \( \tau \). If the government opts for regime shifts, then the value of \( g \) changes from \( g^W \) to \( g^S \). According to equation (4), the government changes its policy regime if and only if

\[ E_\tau \left[ W^{1-\gamma}_T - G \right] > E_\tau \left[ \Phi(C)U^{1-\gamma}_T - G \right]. \]  

(10)

Since regime shifts permanently affect future profitability, the two expectations in equation (10) are determined by different stochastic processes for the aggregate capital \( B_T = \int_0^1 B_i^tdi \). We show the aggregate capital at time \( T \) in the following Lemma.

**Lemma 1.** The aggregate capital at time \( T \), \( B_T = \int_0^1 B_i^tdi \), is given by

\[ B_T = B_\tau e^{\left( \mu + g - \frac{1}{2}\sigma^2 \right)(T-\tau) + \sigma(T-\tau)Z_\tau}, \]  

(11)

in which \( g \equiv g^W \) when there is no policy regime shift, and \( g \equiv g^S \) when there is a policy regime shift.

*Proof. See Lemma 1 in Appendix.*

Plugging the aggregate capital in equation (11) into equation (10), the inequality can be further simplified and provide a decision rule for policy regime shifts in the following Proposition.

**Proposition 1.** The regulation regime changes occur at time \( \tau \) if and only if

\[ c(\tau) < c \]  

(12)
for which
\[ c(\tau) = \log \left\{ e^{(\gamma-1)(g^W-g^S)(T-\tau)} - 1 \right\} > 0. \] (13)

\( p_\tau \) denotes the probability of policy regime shifts at \( \tau \) conditional on information at time \( \tau \)
\[ p_\tau = 1 - \text{Normal}(c(\tau); \hat{c}_\tau, \hat{\sigma}^2_{c,\tau}), \] (14)

for which \( N(x; \hat{c}_\tau, \hat{\sigma}^2_{c,\tau}) \) denotes the c.d.f. of a normal distribution with mean \( \hat{c}_\tau \) and variance \( \hat{\sigma}^2_{c,\tau} \).

Proof. See Proposition 1 in Appendix.

Corollary 1. \( p_{\tau|t} \) denotes the probability of policy regime shifts at \( \tau \) conditional on information at time \( t \)
\[ p_{\tau|t} = 1 - \text{Normal}(c(\tau); \hat{c}_t, \hat{\sigma}^2_{c,t}), \] (15)

for which \( N(x; \hat{c}_t, \hat{\sigma}^2_{c,t}) \) denotes the c.d.f. of a normal distribution with mean \( \hat{c}_t \) and variance \( \hat{\sigma}^2_{c,t} \).

Proof. See Corollary 1 in Appendix.

The decision rule for policy regime shifts is characterized as follows. It causes a regulation change if the perceived environmental cost exceeds a given threshold. Once the cost is above the cutoff, the strong regulation is going to replace the weak regulation when the government perceives the undesirable policy regime under the weak regulation. Given \( \gamma > 1 \), a higher \( \gamma \) implies that households are more risk averse to strong regulation regimes with negative \( g^S \). As a result, the threshold \( c(\tau) \) becomes higher, suggesting a lower probability of shifting to the strong regulation. Moreover, the threshold \( c(\tau) \) depends on the difference between \( g^W \) and \( g^S \). A large difference indicates a costly transition from the weak to strong regulation when the aggregate profitability undergoes a permanent drop. Such an unfavorable economic consequence attenuates the government’s incentive to execute the strong environmental regulation. Therefore, we expect a lower likelihood for environmental policy regime shifts.

2.4 Asset Pricing Implications

In this subsection, we use the following steps to study the asset pricing implications of policy regime shift shocks. First, we show the impact of policy regime shift shocks on the state price of density. Second, we show that firms’ stock prices depend on fundamental shocks and policy regime shift shocks. Finally, we dissect firms’ risk premia attributed to fundamental shocks and policy regime shift shocks, respectively.
Firm $i$’s stock represents a claim on firm $i$’s liquidating dividend at time $T$, which is equal to $B^i_T$. Investors’ total wealth at time $T$ is equal to $B_T = \int_0^1 B^i_t \, dt$. Stock prices adjust such that households hold all of the firm’s stock. In addition to stocks, there is also a zero-coupon bond in zero net supply, which yields a unit payoff at time $T$ with certainty. We use this risk-free bond as the numeraire.\footnote{This assumption is equivalent to assuming a risk-free rate of zero. Such an assumption is innocuous because, without intermediate consumption, there is no intertemporal consumption choice that would clearly identify the interest rate. This modeling choice ensures that interest rate fluctuations do not drive our results.} Under the assumption of market completeness, standard arguments imply that the state price density is uniquely given by

$$\pi_t = \frac{1}{\kappa} E_t \left[ B^{-\gamma}_T \right], \quad (16)$$

for which $\kappa$ is the Lagrange multiplier from the utility maximization problem of the representative household. The market value of stock $i$ is given by the present value of liquidated value at $T$

$$M^i_t = E_t \left[ \frac{\pi_T}{\pi_t} B^i_T \right]. \quad (17)$$

### 2.4.1 State Price Density

Our main focus is on the response of stock prices before regime shift uncertainty is resolved at time $\tau$, for agents learn about the impact of the policy regime as well as the environmental cost under the weak regulation. This learning generates stochastic variation in the posterior mean of $c$, according to equation (8), and the posterior mean represents a stochastic state variable that affects asset prices at time $\tau$. On the other hand, the posterior variance of $c$ varies deterministically over time in equation (9). We first determine the state price of density in the following proposition.

**Proposition 2.** Before the resolution of regime shifts, for $t < \tau$, the state price density is given by

$$\pi_t = B^{-\gamma}_t \Omega_t, \quad (18)$$

for which

$$\Omega_t = e^{\left( -\gamma \mu + \frac{1}{2} \gamma (\gamma+1) \sigma^2 \right) (T-t) - \gamma \sigma^2 (T-\tau) - \gamma \sigma \left( p_{\tau|t} e^{-\gamma \sigma^2 (T-\tau)} + (1 - p_{\tau|t}) e^{-\gamma \sigma^2 (T-\tau)} \right)} \quad (19)$$

**Proof.** See Proposition 2 in Appendix.

The dynamics of the state price of density $\pi_t$ are essential for understanding the source of risks in this economy. An application of Ito’s Lemma to $\pi_t$ determines the stochastic discount factor in Proposition 3.
Proposition 3. The stochastic discount factor (SDF) follows the process

\[
\frac{d\pi_t}{\pi_t} = E_t \left[ \frac{d\pi_t}{\pi_t} \right] - \lambda dZ_t + \lambda_{c,t} d\hat{Z}^c_t,
\]

(20)

for which the price of risk for fundamental shocks denotes

\[
\lambda = \gamma \sigma,
\]

(21)

and the price of risk for uncertainty shocks denotes

\[
\lambda_{c,t} = \frac{1}{\Omega_t} \frac{\partial \Omega_t}{\partial \hat{c}_t} \hat{\sigma}^2_{c,t} \eta^{-1}.
\]

(22)

Proof. See Proposition 3 in Appendix.

Equation (20) shows that the sensitivity of the pricing kernel to fundamental shocks \(\lambda\), and to policy regime shift shocks \(\lambda_{c,t}\), determine the price of risks. Fundamental shocks are represented by the Brownian motion \(dZ_t\), which drives the aggregate fundamentals (profitability) of the economy. The first term of SDF shows that the fundamental shocks affect the SDF in the same way when all parameters are known. The second type of shocks, as introduced from equation (8) to learn about the environmental cost, are unrelated to fundamental shocks (i.e. \(dZ_t \cdot d\hat{Z}^c_t = 0\)). However, policy regime shift shocks affect the allocation of aggregate wealth and, therefore, are priced. Equation (22) indicates that policy regime shift shocks trigger a larger effect on the SDF when the sensitivity of marginal utility to variation in \(\hat{c}_t\) is larger (i.e., \(\partial \Omega_t / \partial \hat{c}_t\) is larger), when updated signals reveal that the environmental cost is larger (i.e., \(\hat{\sigma}_{c,t}\) is larger), and when the accuracy of the uncertainty shocks is larger (i.e., \(\eta^{-1}\) is larger). Above all, the sign of \(\lambda_{c,t}\) is negative. When policy regime shift shocks occur, both the marginal value of wealth and the state price of density increase. Therefore, households are dissuaded from switching to the strong regulation regime; hence, policy regime shift shocks carry a negative price of risk. Overall, a policy regime shift to strong regulation pulls down the aggregate profitability and is viewed as a transition to the bad state in the economy.

2.4.2 Stock Prices and Risk Premia

In this subsection, we present analytical expressions for the level and the dynamics of firm \(i\)’s stock prices, respectively.
Proposition 4. In the benchmark model for $t < \tau$, the stock price for firm $i$ is given by

$$M_t^i = B_t^i \Theta_t^i,$$

(23)

for which

$$\Theta_t^i = e^{(\mu - \gamma \sigma^2)(T - t) + \beta g W(T - t)} \left[ \phi_t e^{\beta g W(T - t)} + (1 - \phi_t) e^{\beta g W(T - t)} \right],$$

(24)

and

$$\phi_t \equiv \frac{p_{\tau|t}}{p_{\tau|t} + (1 - p_{\tau|t}) e^{-\gamma (g W - g S)(T - \tau)}}.$$

(25)

Proof. See Proposition 4 in Appendix.

The dynamics of firm $i$’s stock prices are presented in the following proposition.

Proposition 5. Firm $i$’s realized stock returns at $t < \tau$ follow the process

$$\frac{dM_t^i}{M_t^i} = E_t \left[ \frac{dM_t^i}{M_t^i} \right] + \sigma dZ_t + \sigma_I dZ_t^i + \beta_{M,t}^i d\hat{Z}_c^t,$$

(26)

for which firm $i$’s risk exposure to fundamental and firm-specific shocks denotes $\sigma$ and $\sigma_I$, respectively, and risk exposure to policy regime shift shocks denotes

$$\beta_{M,t}^i \equiv \frac{1}{\Theta_t^i} \frac{\partial \Theta_t^i}{\partial \hat{c}_t} \hat{\sigma}_c^2 \eta^{-1}.$$

(27)

Proof. See Proposition 5 in Appendix.

In equation (27), we show that firm $i$’s realized stock returns contain the risk exposure to fundamental shocks, $\sigma$, firm-specific shocks, $\sigma_I$, and policy regime shift shocks, $\beta_{M,t}^i$. The second term of firm $i$’s realized stock returns shows that all firms in the economy face the same exposure $\sigma$ to fundamental shocks when the parameter $\sigma$ is known. The third term in equation (27) determines firm $i$’s exposure to firm-specific shocks, and is homogeneous to a constant $\sigma_I$. Most importantly, policy regime shift shocks affect firms’ profitability and valuations differently, generating a cross-sectional difference in risk exposures. The last term in equation (27) shows that policy regime shift shocks effect firm $i$’s realized stock returns more strongly when the sensitivity of firm $i$’s valuation $M_t^i$ to variations in $\hat{c}_t$ is larger (i.e., $\partial \Theta_t^i / \partial \hat{c}_t$ is larger), when updated signals reveal that the environmental cost is larger (i.e., $\hat{\sigma}_c^2$ is larger), and when the information about policy regime shocks is more accurate (i.e., $\eta^{-1}$ is larger). Therefore,

$$\beta_{M,t}^i < 0.$$

(28)
which reflects the negative response when policy regime shifts occur. Moreover, we present
the cross-sectional difference in risk exposures to such regime shift shocks in the following
Corollary.

**Corollary 2.** Firm $i$’s exposure to policy regime shift shocks depends on $\xi_i$, which is the
sensitivity of profitability to policy regime shifts.

$$\frac{\partial \beta_{M,t}^i}{\partial \xi^i} < 0.$$  \hspace{1cm} (29)

In equation (29), we show that firm $i$ with a higher $\xi^i$ experiences a larger collapse than
does firm $j$ with a lower $\xi^i$ in realized stock returns. This underlying difference in $\xi^i$ plays an
essential role in determining heterogenous responses to policy regime shifts and in formalizing
the cross-sectional difference in expected stock returns.

In equilibrium, risk premia are determined by the Euler equation that characterizes the
covariance of a firm’s returns with the stochastic discount factor and which is correlated with
fundamental shocks and policy regime shift shocks. The stochastic discount factor is defined
as the growth rate of the state price of density $\pi_t$ and reflects the marginal utility of wealth.
Assets with low payoffs when the state price is high are more undesirable and thus command
higher risk premia. To characterize the risk compensation for fundamental shocks and policy
regime shift shocks, we derive the expressions for the conditional risk premium. In particular,
firm $i$’s expected stock return equals its risk premia

$$E_t \left[ \frac{dM_t^i}{M_t^i} \right] = -\text{Cov}_t \left( \frac{dM_t^i}{M_t^i}, \frac{d\pi_t}{\pi_t} \right) = \sigma \lambda dt - \beta_{M,t}^i \lambda_{c,t} dt.$$  \hspace{1cm} (30)

In equation (30), we show that firm $i$’s risk premia are determined by the exposure to fund-
damental shock and policy regime shifts shock. Furthermore, firm $i$’s risk premia can be
decomposed into the product of the price of risk times firm $i$’s risk exposure, as measured
by the covariance of firm $i$’s realized stock return with shocks. The price of risk of the fund-
damental shock in the stochastic discount factor is constant and depends on the parameter
of risk aversion and the volatility with respect to the fundamental shock. As the price of
risk $\lambda$ is positive, households demand a positive risk premium to invest in securities that are
positively correlated with the fundamental shock.

The risk premium of the policy regime shift shock is in the second term in equation
(30). The price of risk of the regime shift shock, which reflects households’ concerns about
their marginal wealth, is negative. A positive policy regime shift shock lowers aggregate
wealth through an increase in the probability of regime shifts, which leads to a permanent
decrease in aggregate profitability. As a result, a positive uncertainty shock leads to a high
marginal utility of wealth states. We also note that the impact of uncertainty shocks helps
to explain the decline in asset valuations across firms, reflecting a prevailing expectations for
low, future cash flows under regimes with a strong regulation. In particular, high emission
(high $\xi^i$s) firms’ stock prices drop more than those of low emission firms. Therefore, agents
have significant concerns about risk that permanently affects the economy in the future, and
they therefore demand positive compensation for exposure to such uncertainty.

We refer to this premium as the pollution risk premium, so we may emphasize its difference
from risk premium that is driven by fundamental shocks. The cross-sectional asset pricing
implication is captured in the following proposition.

**Proposition 6.** Suppose that there are two firms in the economy: one is a high emission
firm, while the other is a low emission firm. According to equation (30), two firms’ expected
stock returns are denoted as

$$E_t \left[ \frac{dM^H_t}{M^H_t} \right] = \sigma \lambda dt - \beta^H_{M,t} \lambda_{c,t} dt,$$

and

$$E_t \left[ \frac{dM^L_t}{M^L_t} \right] = \sigma \lambda dt - \beta^L_{M,t} \lambda_{c,t} dt,$$

respectively. The long-short portfolio of high versus low emission firms’ expected stock returns
denotes

$$E_t \left[ \frac{dM^H_t}{M^H_t} - \frac{dM^L_t}{M^L_t} \right] = \left[ (-\beta^H_{M,t}) - (-\beta^L_{M,t}) \right] \lambda_{c,t} dt$$

for which $\beta^H_{M,t} < \beta^L_{M,t} < 0$.

*Proof. As discussed earlier, $\xi^L < \xi^H$ and $\partial \beta^i_{M,t} / \partial \xi^i < 0$.*

We make several observations for the long-short portfolio in equation (33) as follows.
First, $\beta^H_{M,t}$ and $\beta^L_{M,t}$ are the risk exposures to uncertainty of the regime shift. When the
regulation regime changes, stock valuations for all firms with positive $\xi^i$s fall, but the stock valuation of firm $H$ with high $\xi^H$ (high emissions) drops more than does that of firm $L$ (low emissions). Therefore, high pollution firms face more exposure to uncertainty shocks. Given the negative price of risk with respect to uncertainty shocks ($\lambda_{c,t} > 0$), investors demand a positive premium to hold high emission firms $H$ over low emission firms $L$. In sum, the pollution risk premium compensates investors for uncertainty in terms of whether strong
regulation would be implemented in the future.

### 2.5 Calibration and Quantitative Model Implications

In this subsection, we calibrate our model at the annual frequency and evaluate its ability to replicate key moments of both real quantities and asset price at the aggregate level. More importantly, we investigate its performance in terms of quantitatively accounting for the pollution premium in the cross-section of expected stock returns. Real quantities refer to the aggregate ROE and book-to-market ratio, while the aggregate asset price refers to the equity premium.

In Table 1, we present a group of calibrated parameter values in our model. We adopt the following calibration procedure to determine a set of sensible parameters. All parameters are grouped into four categories. We determine parameters in the first category by following the previous literature; in particular, we set the relative risk aversion $\gamma$ to be 2 and the volatility to firm-specific productivity shock $\sigma_I$ to be 0.05. These parameters are in line with those in Pástor and Veronesi (2012, 2013). We determine parameters in the second category by matching a set of first and second moments of quantities to their empirical counterparts. The terminal time $T$ is calibrated to be 10, roughly matching an average Compustat firm age of 10 years in our sample. The sample path can be split into two parts when regime shifts occur at the middle $\tau = 5$ between 0 and $T$, without loss of generality. The volatility of the aggregate ROE is set to match 0.10, the second moment of the aggregate ROE in our data.\(^8\) When we determine parameters in the third category, we do not follow an exact one-to-one mapping to the first moment of a specific item in the data; instead, we determine these particular parameters by jointly matching to identify moments in the data: the average of the aggregate ROE, the average of the aggregate book-to-market ratio, changes in ROE driven by regime shifts, and the average of current and future ROE across five quintile portfolios sorted by emissions. Specifically, we estimate the changes in firm-level ROE when firms experience litigation related to environmental issues. As we introduce in the model, $\xi^i$ measures a firm $i$’s vulnerability of profitability to regime shifts, so we choose the distribution of $\xi^i$ between 0 and $T$ to match the current and future ROE in five portfolios. Finally, we let the volatility of the environmental cost $\sigma_c$ equal 0.85, such that the unconditional probability $p_\tau$ is equal to 0.43.$^9$ The volatility of noise parameter $\eta$ is calibrated to be 0.60, approximately matching the equity premium 5.71% per annum. Last but not least, we do not use any information about

---

\(^8\)The aggregate ROE is the cross-sectional average of firm-level ROE in the data, so we can eliminate the firm-specific part.

\(^9\)We do not have the prior value for the probability to regime shifts. As a result, we safely set the probability roughly to be 0.5.
the cross-sectional variation in portfolio returns when we use in our calibration procedure. Instead, we compare the cross-sectional portfolio returns between the data and our simulation to follow our model implication.

[Place Table 1 about here]

We also evaluate the quantitative performance of the model at the aggregate level. In Table 2, we show that our model is broadly consistent with the key empirical features of real quantities and asset price. With respect to real quantities and asset price, our model produces comparable results to our data.

[Place Table 2 about here]

Next, we study the pollution premium at the cross-sectional level. For the purpose of cross-sectional analysis, we make use of several data sources at the micro-level, including (1) firm-level balance sheet data from the Compustat annual files and (2) monthly stock returns from CRSP. In Section 3, we provide additional details regarding our data sources and constructions. Specifically, we set the distribution of exposures $\xi_i$ to regime shifts between 0 and 2, and then simulate 5,000 firms.\(^\text{10}\) In Table 3, we report the average excess returns, book-to-market ratios, current ROEs, and future ROEs across different $\beta_i$, and then compare them with our data.

[Place Table 3 about here]

We document several cross-sectional implications in terms of average returns and firm characteristics in Table 3. First, our model can quantitatively replicate the pattern in the data by generating the upward sloping current ROE but not the downward sloping future ROE across five quintile portfolios sorted by emissions, although both the data and our model feature a flat pattern of book-to-market ratios across portfolios. Second, we use Table 3 to show that our model can generate a pollution premium (i.e., the return spread in the high-minus-low portfolio) as sizable as 4.70%, which is comparable to the 5.52% we obtain from our data in Section 4.1. To generate the pollution premium, we identify a key mechanism: high emission firms’ cash flows are more vulnerable to regime shifts from weak to strong regulations, so they face higher risk exposures to regime shift shocks. Hence, investors demand higher expected returns to hold high emission firms’ stocks.

\(^{10}\)The range of $\xi_i$ implies that the cross-sectional mean is equal to 1.
3 Construction of Emissions

3.1 The Data and Construction

We construct firm-level emissions of U.S. public companies by collecting plant-level chemical pollutants data from the Toxic Release Inventory (TRI) database constructed by the United States Environmental Protection Agency (EPA). The database contains the following detailed information on all U.S. chemical emissions from 1986 to 2014: report year, level of chemical pollutants (pounds), name of chemical categories, location fips code, and company names. The TRI database is a publicly available database operated by EPA since 1986. In response to incomplete coverage and measurement errors in the early period, we use the TRI data from 1990 to 2014.

Unlike accounting reports or corporate taxation, the pollutants we report are self-reported; as a result, and importantly, this pollutant data lacks reasonable verifications from third parties such as auditors and IRS. Hence, there is no mechanism of enforcement to inhibit firms from intentionally under-reporting their pollutants, which could undermine the reliability of chemical pollutant data. Therefore, the chemical pollutants that we obtain and report from the database are inevitably subject to measurement errors.

Our sample consists of firms in the intersection of Compustat, CRSP (Center for Research in Security Prices), the TRI database, and Capital IQ. We obtain accounting data from Compustat and stock data from CRSP. Our sample firms include those with non-missing TRI data and non-missing SIC codes, and those with domestic common shares (SHRCD = 10 and 11) trading on NYSE, AMEX, and NASDAQ. We identify firms in our sample that were involved in litigations from the Key Developments in Capital IQ. Following the literature, we exclude finance firms that have four-digit standard industrial classification (SIC) codes between 6000 and 6999 (e.g., finance, insurance, trusts, and real estate sectors) and firms with negative book value of equity. To mitigate backfilling bias, we require firms to be listed on Compustat for two years before we include them in our sample. All firm-level variables, except emission measures, are from Compustat, unless otherwise noted. Moreover, we use the life expectancy data of Wang, Schumacher, Levitz, Mokdad, and Murray (2013), county-level unemployment rate and population data, and state-level personal income per

---

11 The U.S. congress passed the Community Right to Know Act (EPCRA) in 1986 in response to public concerns over the release of toxic chemicals from several environmental accidents, both in domestic and overseas. EPCRA entitles residents in their respective neighborhoods to know the source of detrimental chemicals, especially for their potential impacts on human health from routes of exposure. EPCRA requires a compulsory disclosure from each firm on its chemical releases to the environment with emission that exceeds the amounts of listed toxic substances. Following the EPCRA, EPA constructs TRI to track and supervise certain classifications of toxic substances from chemical pollutants that endanger human health and the environment.
capita data from the Federal Reserve Economic Data (FRED), which is maintained by the Federal Reserve in St. Louis.

Finally, we collect news about firms involved in litigations from Capital IQ. More specifically, Capital IQ covers information with material impact on the market value of securities, including executive changes, M&A rumors, changes in corporate guidance, delayed filings, SEC inquiries, and litigations. We search these firms’ new coverage in capital IQ using the following keyword phrases: "lawsuit", "litigation", "penalty", and "settlement". We then manually identify those firms involved in litigations related to violations of environmental regulations.

3.2 Measures of Emissions

As we mentioned in our introduction to the TRI database, the EPA reports levels of chemical pollutants at the county level for each year. We sum the reported chemical pollutants across all counties reported by a firm in a given year to measure firm-level chemical pollutants in millions of pounds and then scale them by book equity in millions of dollars to obtain our first empirical proxy for emission intensity: ”Simple Emissions.” However, using simple summation of chemical pollutants over counties ignores the heterogeneous toxicities of different chemical categories; in short, some chemical categories may be more lethal to human health than others. Therefore, we use an approach to calculate the toxicity degrees for each chemical category to obtain ”Toxicity-adjusted Emissions” by weighting emissions with toxicity degrees. In particular, we run county-year panel regressions for each chemical category at expanding windows: \[ \Delta \text{Life\_Exp}_{it} = a + b^j \times \text{Chem}^j_{it} + X_{it}b + \theta_t + c_i + \varepsilon_{it}, \text{ for } j = 1, ..., J, \]

for which $\Delta \text{Life\_Exp}_{it}$ is the change in life expectancy in county $i$ in year $t$, $\text{Chem}^j_{it}$ is the level of chemical pollutants for the category $j$, and $X_{it}$ are control variables for economic fundamentals, including county-level employment rates, population, and state-level personal income per capita. We also control for county fixed effects $c_i$ and year fixed effects $\theta_t$. Standard errors are clustered at the county level. We proxy $b^j$ for the toxicity degree for a given chemical category $j$. A lower estimate of $b^j$ suggests that the category $j$ is more hazardous to human beings.\[13\]

\[12\]The first window is 1990 only and is used to estimate toxicity degrees for 1991. The second window is 1990-1992, which we use to estimate toxicity degrees for 1992; we follow a similar procedure until 2010. Given that we use the life expectancy data up to 2010, toxicity degrees from 2011 to 2014 are based on the estimates that we obtain in 2010.

\[13\]There are currently more than 650 chemicals categories reported in the TRI database; however, not all of these categories exist at the commencement of the TRI program. Only 586 chemical categories are available
The estimation for $b_j$ may result in some outlier coefficients of very negative or very positive numbers, which cannot be used to construct our toxicity-weighted emissions. Thus, for each year, we sort all categories with no-missing and negative estimates into five groups based on the estimated $b_j$, and then assign a score of 6 to the lowest quintile, 5 to the second-lowest quintile, 4 to the third quintile, 3 to the second-highest quintile, and 2 to the highest quintile.\textsuperscript{14} All chemical pollutants are supposed to have negative impacts on human health; however, owing to measurement errors and data limitations, we may have positive coefficients for some $b_j$, or too few observations to estimate $b_j$. We thus assign a score of 1 to those categories. Such score assignment ensures that our weighting is less affected by outliers. Finally, we calculate a firm’s “Toxicity-adjusted Emissions” as the weighted sum by multiplying the level of pollutants produced by a firm in a category with the score of that category.

In Panel A of Table IA.1 in the Internet Appendix, we report the summary statistics of all chemical categories and the time-series average of assigned scores based on the estimated coefficient from equation (34). Some chemical categories have higher scores than other categories. For example, "BIS(TRIBUTYL Tin) OXIDE" is a substance highly concentrated in liver and kidney, and remains unknown for its impact on human health. Meanwhile, "3 3'-DIMETHOXYBENZIDINE" is confirmed to trigger cancer, based on evidence from animal experimentation. Moreover, physical contact (e.g., inhalation, ingestion) with "TRIBUTYL Tin METHACRYLATE" causes severe injury or even death. In Panel B, we report the companies in our sample with the most toxic chemical pollutants based on hazardous score in Panel A. Products from these firms are mainly related to agriculture, chemical, energy, food, and steel and coal industries. Agrium, Inc, American Vanguard Corp, and Scotts Mirable-gro Co, which all focus on garden and lawn care, produce chemical fertilizers. Also, Akzo Nobel Nv produces products related to chemical paints and coatings. In addition, Albemarle Corp is the leader in the lithium battery market. Finally, Dow Chemical and DuPont, which are two giants in the chemicals industry, and both have significant presence in the U.S.

\subsection*{3.3 Summary Statistics}

In addition to simple and toxicity-adjusted emissions, we consider the following variables: market capitalization (i.e., size), the book-to-market ratio (B/M), investment rate (I/A), asset growth (AG), return on equity (ROE), R&D intensity (R&D/AT), organization capital in our sample. Thus, the changes in chemicals categories could cause inconsistency when we estimate toxicity weights across years. We use both simple emissions and toxicity-adjusted emissions in our empirical analysis to ensure the robustness of our conclusion.

\textsuperscript{14}In 1990, given no prior observations, we treat all chemical categories equally by assigning a score of 1 to all categories.
ratio (OC/AT), and book leverage.

In Table 4, we report pooled summary statistics and correlation between emission measures in year t-1 and other characteristics that are known to the public at the end of June of year t. ME is market capitalization (measured in millions USD) at the end of June of year t. Book-to-market ratio (B/M) is the ratio of book equity of the fiscal year ending in year t-1 to market capitalization at the end of year t-1. Investment rate (I/A) is capital expenditure in fiscal year t-1 divided by lagged total assets at the end of fiscal year t-2. Asset growth (AG) is the change in total assets in fiscal year t-1 divided by lagged total assets. Return on equity (ROE) is income before extraordinary items plus depreciation expenses in fiscal year t-1 scaled by lagged book equity. R&D/AT is the R&D expenses capital\textsuperscript{15} divided by total assets in fiscal year t-1. OC/AT is the organization capital divided by total assets in fiscal year t. In Panel A, we report the pooled mean, median, standard deviation (Std), minimum (Min), 25\textsuperscript{th} percentile (P25), medium, 75\textsuperscript{th} percentile (P75), and Maximum (Max). Obs denotes the valid number of observations for each variable. We have a total 158,344 firm-year observations with non-missing simple and toxicity-adjusted emissions. The averages of simple emissions and toxicity-adjusted emissions are 0.031 and 0.049, respectively, suggesting that one thousand dollars of book equity are associated with 12.96 to 16.94 million pounds of emissions.

[Place Table 4 about here]

\section{4 Empirical Analysis}

In this section, we provide novel empirical evidence for the positive relation between toxic emissions and the cross-section of stock returns. We first show that emissions positively predict cross-sectional expected stock returns in portfolio sorts. We then provide testable implications to support our model mechanisms. Furthermore, we perform a battery of asset pricing factor tests to show that such a relation is literally unaffected by known return factors for other systematic risks. Finally, we investigate the joint link between emissions and other firm-level characteristics on one hand and future stock returns in the cross-section on the other, using Fama and MacBeth (1973) regressions as a valid cross-check for the positive relation between emissions and stock returns.

\textsuperscript{15}We follow Chan, Lakonishok, and Sougiannis (2001) to accumulate R&D expenditures over the most recent five fiscal years at a 20\% depreciation rate.
4.1 The Pollution Premium and Firm Characteristics

To investigate the link between simple (toxicity-adjusted) emissions and future stock returns in the cross-section, we construct five portfolios sorted on a firm’s current simple (toxicity-adjusted) emissions and report the portfolio’s post-formation average stock returns. We construct the simple (toxicity-adjusted) emissions at an annual frequency as described in Section 3. At the end of June of year $t$ from 1992 to 2015, we rank firms by simple emissions (toxicity-adjusted emissions) relative to their industry peers and construct portfolios as follows. At the end of each June from 1992 to 2015, we sort all NYSE\textsuperscript{16} firms with positive simple (toxicity-adjusted) emissions in year $t-1$ into five groups from low to high within the corresponding 48 industries, according to Fama and French (1997). As a result, we have industry-specific, NYSE-based breaking points for quintile portfolios for each June. We then assign all other non-NYSE firms with positive simple emissions (toxicity-adjusted emissions) in year $t-1$ into these portfolios. Thus, the low (high) portfolio contains firms with the lowest (highest) emissions in each industry. To examine the emission-return relation, we form a high-minus-low portfolio that takes a long position in the high emission portfolio and a short position in the low emission portfolio.

After forming the six portfolios (from low to high and high-minus-low), we calculate the value-weighted monthly returns on these portfolios over the next twelve months (July in year $t$ to June in year $t+1$). In Panel A (Panel B) of Table 5, the top row presents the annualized average excess stock returns ($\text{E}[R]-R_f$, in excess of the risk free-rate), standard deviations, and Sharpe ratios of the five portfolios sorted on simple (toxicity-adjusted) emissions. With Table 5, we show that, consistent with our model, a firm’s emissions forecast stock returns. Firms with currently high emissions earn subsequently lower returns, on average, than firms with currently high emissions. The difference in returns is economically large and statistically significant.

In both Panels A and B, we find that the average excess returns on the first five portfolios strictly increase with simple (toxicity-adjusted) emissions. From low to high quintiles, the average excess returns are 7.31%, 7.82%, 7.99%, 8.46%, and 12.84%, respectively. In addition, the average excess return on the high-minus-low portfolio is 5.52% with statistical significance at the 1% level. In Panel B, the average excess return from the low portfolios to the high portfolios are 6.90%, 8.31%, 7.99%, 7.89%, and 12.77%, respectively. The average excess return on the high-minus-low portfolio is 5.87% with statistical significance at the 1% level. The average return spread (i.e., the average return on the high-minus-low portfolio) is more

\textsuperscript{16}Unlike the Compustat-CRSP merged sample, more than one third of firms in our sample are NYSE-listed firms because we require non-missing and non-zero emission data. Nevertheless, when we sort by NYSE breaking points, we obtain results that reflect an evenly distributed number of firms in five portfolios (see Table 5).
than 3 standard errors from zero. Across the two sets of average returns, the Sharpe ratio of the portfolio of firms with high simple emissions (toxicity-adjusted emissions) is more than 1.7 (1.8) times larger than the Sharpe ratio of the portfolio of firms with low simple emissions (toxicity-adjusted emissions).

[Place Table 5 about here]

We then examine the time-series pattern of the returns on the high-minus-low portfolio, which is our proxy for pollution premium. In Figure 1, we plot the cumulative returns of the high-minus-low portfolio from July of 1992 to December of 2015. The portfolio’s cumulative returns reveal a clear steady upward trend. While there are some drops in the cumulative returns, they do not overlap with economic recessions (denoted by the shaded areas). As a result, the positive emission-return relation that we find appears to be a fairly persistent pattern.

A natural concern for the measure of simple (toxicity-adjusted) emissions is whether the simple (toxicity-adjusted) emissions merely captured by other firm characteristics are known to predict cross-sectional stock returns. In Table 6, we report firm characteristics across quintile portfolios sorted on simple (toxicity-adjusted) emissions. There are small dispersions for size, book-to-market, and investment rate. From low to high quintile portfolios, we observe upward sloping patterns in profitability (ROE), R&D intensity, and leverage, but downward sloping patterns in asset growth and organization capital ratio.

[Place Table 6 about here]

4.2 Further Tests for Model Assumptions and Implications

We provide direct empirical evidence to support our model assumption on firms’ profitability, and we justify that firms with high emissions face high probabilities to trigger litigations that negatively affect those firms’ profitability. Moreover, we show test results consistent with our model predictions. Specifically, we measure the perceived possibility of policy regime shift shocks using the log difference (i.e., the growth rate) of the total number of firms that report their toxic emissions, temperature, and rainfall.17 These three growth rates reflect the perceived likelihood for changes in the government’s perceived environmental cost that could reshape the government’s policy regime.

17The use of temperature and rainfall is motivated by Bansal, Kiku, and Ochoa (2017). These data are collected from The World Bank’s Climate Change Knowledge Portal.
4.2.1 Future Profitability

In our model, firms’ profitability drops when regulation tightens, as we show in equation (2). To test this model assumption, we focus on simple emissions and use three measures of perceived probability of policy regime shifts “Shocks.” We express our specification as follows

\[ ROE_{i,t+5} = a + b_1 \times Emissions_{i,t} + b_2 \times Shocks_t + b_3 \times Emissions_{i,t} \times Shocks_t + c \times Controls_{i,t} + \varepsilon_{i,t}, \]

for which \( ROE_{i,t+5} \) is firm \( i \)'s ROE in \( t + 5 \), \( Emissions_{i,t} \) denotes firm \( i \)'s simple or toxicity-adjusted emissions in year \( t \), and \( Shocks_t \) denotes one of the three measures for the perceived probability of shocks in year \( t \): "Disclosure,” "Temperature,” and "Rainfall.” We control for a firm’s fundamentals, including size, book-to-market ratio (B/M), investment rate (I/A), asset growth (AG), return on equity (ROE), R&D intensity (R&D/AT), organization capital ratio (OC/AT), and book leverage in year \( t \) and industry fixed effect. Standard errors are clustered at the firm level.

[Place Table 7 about here]

We find that, consistent with our model, the estimated coefficients for the interaction term \( \hat{b}_3 \) are negatively significant across different measures of signal shocks. On the other hand, the estimated coefficients \( \hat{b}_1 \) and \( \hat{b}_2 \) are insignificantly different from zero when we control for the interaction term. Our interpretation for this pattern is that all firms’ profits are subject to negative exposure to policy regime shocks. However, different firms have different exposure. The negatively significant coefficient \( \hat{b}_3 \) suggests that firms with higher toxic emissions experience more profitability decline when policy regimes shift. Overall, we obtain robust and consistent results that policy regime shifts hurt firms’ future profitability.

4.2.2 Future Litigations Related to Environmental Issues

In this subsection, we justify the finding in our previous subsection and show that high emission firms are subject to more environmental litigation probabilities when a policy regime shifts. We use firm-level litigation information, including a firm’s disclosure of material information about legal authority and enforcement or lawsuits relevant to environmental issues from Capital IQ, as a proxy of the consequences of policy regime shifts.\(^ {18} \) We test this prediction by estimating

\[ N_{i,t+5} = a + b_1 \times Emissions_{i,t} + c \times Controls_{i,t} + \varepsilon_{i,t}, \]

\(^ {18} \)In our model, the implied regime shift is irreversible and occurs only once. However, policy regime shifts in reality might occur from time to time.

24
for which $N_{i,t+5}$ refers to firm $i$’s future litigation status, which is defined as a binary variable and reflects whether a firm is involved in litigations in the next five years or which is defined as a count variable and reflects the total involved litigations in the next five years. Since the first measure is binary we estimate equation (36) using a Probit regression; since the second is a count variable, we estimate equation (36) using a Poisson count and negative binomial regression, respectively. We report the estimated coefficients in Table 8.

We find that simple emissions in all predictive regressions are positively significant to predict future litigations in the left panel of Table 8. We also show similar result are shown from the predictive regressions by toxicity-adjusted emissions in the right panel of Table 8. Therefore, our empirical evidence confirms that high emission firms will be involved in more environmental litigations from strong regulation regimes when a policy regime shifts.

4.2.3 Realized Stock Returns

In this subsection, we show results consistent with Proposition 5 and Corollary 2. In our model, realized stock returns decrease with policy regime shift shocks, as we show in equation (27). More significant for our model implications, however, is that high emission firms carry more negative exposure with respect to policy regime shift shocks. To support these model implications, we explore the relation between realized stock returns and three measures for the probability of regime shifts (“Shocks”) as defined earlier:

$$R_{i,t} - R_{f,t} = a + b_1 \times Emissions_{i,t} + b_2 \times Shocks_t + b_3 \times Emissions_{i,t} \cdot Shocks_t + c \times Controls_{i,t} + \varepsilon_{i,t},$$

(37)

for which $R_{i,t} - R_{f,t}$ is firm $i$’s stock return in calendar year $t$, $Emissions_{i,t}$ denotes firm $i$’s simple or toxicity-adjusted emissions in year $t$, and $Shocks_t$ denotes the perceived probability of shocks in year $t$. We estimate this equation via the Fama-Macbeth regression and report our estimated coefficients, along with Newey-West standard errors, in Table 9.

In Table 9, we show that the estimated coefficients of $\hat{b}_3$ are all negatively significant across different specifications, consistent with Proposition 5 and Corollary 2. The negatively significant coefficient $\hat{b}_3$ suggests that high emission firms’ market value decreases more than low emission firms’ when the policy regime shifts. As a result, we observe drops in firms’ realized stock returns.
4.3 Asset Pricing Factor Tests

We also investigate the extent to which the variation in the average returns of the emission-sorted portfolios can be explained by exposure to standard risk factors proposed by the Fama and French (2015) five-factor model or the Hou, Xue, and Zhang (2015) q-factor model. \(^{19}\)

To test the standard risk factor models, we preform time-series regressions of emissions sorted portfolios’ excess returns on the Fama and French (2015) five-factor model (the market factor-MKT, the size factor-SMB, the value factor-HML, the profitability factor-RMW, and the investment factor-CMA) in Panel A and on the Hou, Xue, and Zhang (2015) q-factor model (the market factor-MKT, the size factor-SMB, the investment factor-I/A, and the profitability factor-ROE) in Panel B, respectively. Such time-series regressions enable us to estimate the betas (i.e., risk exposures) of each portfolio’s excess return on various risk factors and to estimate each portfolio’s risk-adjusted return (i.e., alphas in %). We annualize the excess returns and alphas in Table 10.

[Place Table 10 about here]

As we show in Table 10, the risk-adjusted returns (intercepts) of the simple (toxicity-adjusted) emissions sorted high-minus-low portfolio remain large and significant, ranging from 5.25 (5.30)% for the Fama and French (2015) five-factor model in Panel A to 5.06 (5.15)% for the Hou, Xue, and Zhang (2015) q-factor model in Panel B, and these intercepts are 3 standard errors above zero, which the t-statistics is far above 1% statistical significance level. Second, the alpha implied by the Fama-French five-factor model is slightly higher than the the simple (toxicity-adjusted) emissions spread (i.e., the return on the high-minus-low portfolio) in the univariate sorting (Table 5), while the alpha implied by the HXZ q-factor model remains comparable to the long-short portfolio sorted on simple (toxicity-adjusted) emissions. Third, the return on the high-minus-low portfolio has insignificantly negative betas with respect to both the Fama and French (2015) five-factor model and to the Hou, Xue, and Zhang (2015) q-factor model. The high-minus-low portfolio based on simple emissions presents negative loadings on market and size for the Fama-French five-factor model (Panel A) and the HXZ q-factor model (Panel B). In summary, results from asset pricing tests in Table 10 suggest that the cross-sectional return spread across portfolios sorted on simple (toxicity-adjusted) emissions cannot be explained by either the Fama and French (2015) five-factor model or the HXZ q-factor model (Hou, Xue, and Zhang (2015)). Hence, common risk factors cannot explain the higher returns associated with pollution. In the next section,

\(^{19}\)The Fama and French factors are downloaded from Kenneth French’s data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). We thank Kewei Hou, Chen Xue, and Lu Zhang for kindly sharing the Hou, Xue, and Zhang factors.
we go beyond factor regressions and control for multiple characteristics simultaneously by running cross-sectional regressions.

4.4 Cross-Sectional Return Predictability Regressions

For robustness, we also investigate the predictive ability of emissions for the cross-sectional stock returns using Fama-MacBeth cross-sectional regressions (Fama and MacBeth (1973)). This analysis allows us to control for an extensive list of firm characteristics that predict stock returns and to verify whether the positive emission-return relation is driven by other known predictors at the firm level. This approach is preferable to the portfolio tests, as the latter requires the specific breaking points to sort firms into portfolios and also requires us to select the number of portfolios. Also, it is difficult to include multiple sorting variables with unique information about future stock returns by using a portfolio approach. Thus, Fama-MacBeth cross-sectional regressions provide a reasonable cross-check.

We run standard firm-level, Fama-MacBeth cross-sectional regressions to predict stock returns using lagged firm-level emissions after we control for other characteristics. The specification of regression is as follows:

\[ R_{i,t+1} - R_{f,t+1} = a + b \times Emissions_{i,t} + \gamma \times Controls_{i,t} + \epsilon_{it}. \]

Following Fama and French (1992), we take each month from July of year t to June of year t+1, and we regress monthly returns of individual stock returns (annualized by multiplying 12) on emissions of year t-1, different sets of control variables that are known by the end of June of year t, and industry fixed effects. Control variables include the natural logarithm of market capitalization at the end of each June (Size), the natural logarithm of book-to-market ratio (B/M), investment rate (I/A), asset growth (AG), return on equity (ROE), R&D intensity (R&D/AT), organization capital ratio (OC/AT), book leverage (Leverage), and industry dummies based on Fama and French (1997) 48 industry classifications. All independent variables are normalized to a zero mean and a one standard deviation after winsorization at the 1th and 99th percentile to reduce the impact of outliers; we also adjust all independent variables for standard errors by Newey-West adjustment.

In Table 11, we report the results from cross-sectional regressions performed at a monthly frequency. The reported coefficient is the average slope from monthly regressions, and the corresponding t-statistic is the average slope divided by its time-series standard error. We annualize the slopes and standard errors in Table 11.
The results of Fama-Macbeth regression are consistent with the results of portfolio sorted on emissions. In Specification 1 (3), simple (toxicity-adjusted) emissions significantly and positively predict future stock returns with a slope coefficient of 7.18 (6.80), which is 2.55 (2.66) standard errors from zero. This finding implies that a one-standard-deviation increase in emissions leads to a significant increase of 7.18 (6.80)% in the annualized stock return. The difference in average simple (toxicity-adjusted) emissions between firms in the top and bottom quintile is around 0.68 (0.69) standard deviations. The coefficient in Column 1 (3) implies a difference in the annualized return to 4.93 (4.72)%, which is slightly lower than the high-minus-low portfolio return of 5.52 (5.87)% that we report in Table 5. The Fama-Macbeth regressions suggest that emissions positively predict average returns. Such a regression weighs each observation equally, and thus places substantial weight on small firm. However, our finding for the pollution premium is mainly based on value-weighted rather than equal-weighted portfolios. Therefore, the difference between valued- and equal-weighted portfolios reflects the discrepancy between the implied return from the Fama-Macbeth regression and the valued-weighted portfolio return.

From Specification 2 (4), simple (toxicity-adjusted) emissions positively predict stock returns with statistically significant slope coefficients when we further control for size, book-to-market ratio, investment rate, asset growth, ROE, R&D intensity, organizational capital ratio, and leverage. Of note, the slope of simple (toxicity-adjusted) emissions remains positive and significant for Specification 2 (4) after we include all the regressors. Overall, Table 11 suggests that the positive emission-return relation cannot be attributed to other known predictors and that simple (toxicity-adjusted) emissions have an unique return predictive power.

5 Conclusion

The awareness of environmental protection has surged over the past several decades. This paper investigates the implications of pollution on the cross-section of stock returns. We use chemical emissions reported to the Environmental Protection Agency (EPA) to measure firms’ annual toxic release. A long-short portfolio constructed from firms with high versus low toxic emission intensity relative to their industry peers generates an average excess return of around 5.52% per year. The return spread cannot be explained by existing risk factors, including the Fama-French five-factor model (Fama and French (2015)) and the HXZ q-factor model (Hou, Xue, and Zhang (2015)). Fama and MacBeth (1973) regressions provide a valid cross-check for the positive relation between toxic emissions and stock returns. We also find a negative relation between toxic emissions and future profitability as measured by ROE.
To explain our empirical finding of a pollution premium, we develop a general equilibrium asset pricing model in which firms’ cash flows face the uncertainty of regime shifts in emission regulation policy. In our model, a government (i.e., social planner) makes an optimal decision between a strong or weak emission regulation regime by maximizing an investor’s welfare based on such a trade-off, as a social planner would do. In particular, we find that the government optimally replaces a weak regulation regime by a strong one if the pollution cost is perceived to be sufficiently higher. Since high emission (“dirty”) firms’ profitability is more negatively affected than that of low emission (“clean”) firms upon a regime shift from weak to strong regulation, high emission (“dirty”) firms are more exposed to regulation regime shift risks and, thus, earn higher average excess returns as risk premia.

Further empirical analyses provide supportive evidence to our model assumptions and implications. First, all firms’ future profits decrease with the perceived probability of policy regime shifts, and high emission firms suffer more. Second, we verify the channel for the reduced profitability by showing that high emission firms are involved in more environmental litigations when a policy regime shifts. Last and most importantly, we show that high emission firms’ market value drops significantly when a policy regime shifts.
References


Hong, Harrison, Frank Weikai Li, and Jiangmin Xu, 2016, Climate risks and market efficiency, Technical report, National Bureau of Economic Research.


Lochstoer, Lars A, 2009, Expected returns and the business cycle: Heterogeneous goods and

Loualiche, Erik, 2016, Asset pricing with entry and imperfect competition.

Pástor, L’uboš, and Pietro Veronesi, 2012, Uncertainty about government policy and stock

Pástor, L’uboš, and Pietro Veronesi, 2013, Political uncertainty and risk premia, *Journal of
Financial Economics* 110, 520–545.


van Bingbergen, Jules, 2016, Good-specific habit formation and the cross-section of expected

Wang, Haidong, Austin E Schumacher, Carly E Levitz, Ali H Mokdad, and Christopher JL
Murray, 2013, Left behind: widening disparities for males and females in us county life

Yogo, Motohiro, 2006, A consumption-based explanation of expected stock returns, *The

Figure 1. Calendar-Time Cumulative Returns of the High-minus-Low Portfolios

Cumulative returns are computed for the high-minus-low portfolios sorted by simple and toxicity-adjusted emissions. We plot the time-series of these cumulative returns. The shaded bands are labeled as recession periods, according to NBER recession dates. The sample period is July 1992-Dec 2015.

![Cumulative Returns](image-url)
Table 1. Parameter Choices

This table reports the parameter values used in the simulations. The parameters are: regime shifts uncertainty $\sigma_c$ in equation (5), volatility of noise $\eta$ in equation (6), $\mu$, $\sigma$, and $\sigma_I$ from equation (1), final date $T$, time $\tau$ of the policy decision, and risk aversion $\gamma$. $\sigma_c$ and $\eta$ equate the real quantities and equity premium. All variables except for $\gamma$ are reported on an annual basis.

<table>
<thead>
<tr>
<th>$\sigma_c$</th>
<th>$\eta$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\sigma_I$</th>
<th>$T$</th>
<th>$\tau$</th>
<th>$\gamma$</th>
<th>$g^W$</th>
<th>$g^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>0.60</td>
<td>0.20</td>
<td>0.10</td>
<td>0.05</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>0.02</td>
<td>-0.06</td>
</tr>
</tbody>
</table>
Table 2. Aggregate Moments

This table reports aggregate quantities (Panel A) and asset price (Panel B) in the model and data. Aggregate quantities refer to aggregate ROE and book-to-market ratio, and aggregate asset price refers to the equity premium in annual frequency.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Real Quantities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROE</td>
<td>0.23</td>
<td>0.22</td>
</tr>
<tr>
<td>B/M</td>
<td>0.38</td>
<td>0.40</td>
</tr>
<tr>
<td><strong>Panel B: Asset Price</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R_m]-R_f$ (%)</td>
<td>5.71</td>
<td>4.70</td>
</tr>
</tbody>
</table>
Table 3. Portfolios, Firm Characteristics, and Model Comparison

This table reports time-series averages of the cross-sectional averages of firm characteristics across five portfolios sorted on emissions. Panel A reports the five quintile portfolio sorted from the data, as mentioned in Section 4.1. Panel B reports five quintile portfolios sorted from the simulation. All returns are annualized.

<table>
<thead>
<tr>
<th>Variables</th>
<th>L</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>H</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>E[R]-R_f (%)</td>
<td>7.31</td>
<td>7.82</td>
<td>7.99</td>
<td>8.46</td>
<td>12.84</td>
<td>5.52</td>
</tr>
<tr>
<td>ROE</td>
<td>0.17</td>
<td>0.20</td>
<td>0.21</td>
<td>0.27</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>ROE_{t+5}</td>
<td>0.35</td>
<td>0.19</td>
<td>0.25</td>
<td>0.20</td>
<td>0.17</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>L</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>H</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>E[R]-R_f (%)</td>
<td>2.51</td>
<td>3.57</td>
<td>4.68</td>
<td>5.90</td>
<td>7.21</td>
<td>4.70</td>
</tr>
<tr>
<td>ROE</td>
<td>0.20</td>
<td>0.21</td>
<td>0.22</td>
<td>0.23</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>ROE_{t+5}</td>
<td>0.20</td>
<td>0.18</td>
<td>0.17</td>
<td>0.16</td>
<td>0.15</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Summary Statistics

This table presents summary statistics for the firm-year-month sample. Simple emissions are measured as the simple summations of chemical pollutants over counties in year t-1, and then divided by the book value of equity at the end of fiscal year t-1 at the firm-year level. Toxicity-adjusted emissions are measured as the hazardous-weighted summation of chemical pollutants over counties in year t, and then divided by the book value of equity at the end of fiscal year t-1 at the firm-year level. ME is market capitalization (millions $) at the end of June of year t. \(B/M\) is the ratio of the book equity of fiscal year ending in year t-1 to market capitalization at the end of year t-1. \(I/A\) is capital expenditure (item CAPX) dividend by lagged total assets at the end of fiscal year t-2. Asset growth (AG) is the change in total assets in year t-1 divided by lagged total assets. Return on equity (ROE) is income before extraordinary items plus interest expenses in year t-1 scaled by lagged book equity. R&D/AT is the summation of R&D expenses by inventory method over the previous five years divided by lagged total assets. OC/AT is the summation of general administrative expenses by inventory method over the previous five fiscal years divided by lagged total assets. We report the pooled mean, median, standard deviation (Std), minimum (Min), 25th percentile (P25), medium, 75th percentile (P75), and Maximum (Max). Obs denotes the valid number of observations in each variable. The sample period is 1991-2014.

<table>
<thead>
<tr>
<th>Summary Statistics</th>
<th>Simple Emissions</th>
<th>Toxicity-adjusted Emissions</th>
<th>ME</th>
<th>B/M</th>
<th>I/A</th>
<th>AG</th>
<th>ROE</th>
<th>R&amp;D/AT</th>
<th>OC/AT</th>
<th>Leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>12.96</td>
<td>16.94</td>
<td>98,146.06</td>
<td>0.38</td>
<td>0.06</td>
<td>1.11</td>
<td>0.23</td>
<td>0.10</td>
<td>0.49</td>
<td>0.36</td>
</tr>
<tr>
<td>Std</td>
<td>55.75</td>
<td>53.02</td>
<td>27,931.47</td>
<td>0.64</td>
<td>0.06</td>
<td>0.39</td>
<td>2.17</td>
<td>0.11</td>
<td>0.42</td>
<td>0.22</td>
</tr>
<tr>
<td>Min</td>
<td>0.00</td>
<td>0.00</td>
<td>0.44</td>
<td>0.00</td>
<td>0.00</td>
<td>0.16</td>
<td>-126.86</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>P25</td>
<td>0.00</td>
<td>0.00</td>
<td>309.85</td>
<td>0.33</td>
<td>0.03</td>
<td>0.99</td>
<td>0.05</td>
<td>0.00</td>
<td>0.23</td>
<td>0.20</td>
</tr>
<tr>
<td>Median</td>
<td>0.83</td>
<td>1.36</td>
<td>1,261.27</td>
<td>0.52</td>
<td>0.05</td>
<td>1.06</td>
<td>0.11</td>
<td>0.03</td>
<td>0.44</td>
<td>0.35</td>
</tr>
<tr>
<td>P75</td>
<td>4.60</td>
<td>7.72</td>
<td>4,820.41</td>
<td>0.8</td>
<td>0.08</td>
<td>1.15</td>
<td>0.17</td>
<td>0.08</td>
<td>0.71</td>
<td>0.50</td>
</tr>
<tr>
<td>Max</td>
<td>230.06</td>
<td>277.95</td>
<td>524,351.6</td>
<td>24.75</td>
<td>1.83</td>
<td>12.89</td>
<td>70.38</td>
<td>3.49</td>
<td>5.06</td>
<td>1.00</td>
</tr>
<tr>
<td>Obs</td>
<td>158,344</td>
<td>158,344</td>
<td>141,397</td>
<td>141,284</td>
<td>155,516</td>
<td>156,813</td>
<td>158,291</td>
<td>158,344</td>
<td>158,344</td>
<td>158,098</td>
</tr>
</tbody>
</table>
Table 5: Portfolios Sorted on Emissions

This table shows average excess returns for five portfolios sorted on simple emissions portfolios (Panel A) and on toxicity-adjusted emissions portfolios (Panel B) relative to their industry peers, for which we use the Fama-French 48 industry classifications and rebalance portfolios at the end of every June. The results reflect monthly data, for which the sample period is from July 1992 to December 2015 and excludes financial industries. We report average excess returns over the risk-free rate $E[R]-R_f$, standard deviations Std, and Sharpe ratios SR across five portfolios in Panel A and Panel B. Standard errors are estimated by using the Newey-West correction. We include t-statistics in parentheses and annualize portfolio returns multiplying by 12. All portfolios returns correspond to value-weighted returns by firm market capitalization. All returns, standard deviations, and Sharpe ratios have been annualized.

<table>
<thead>
<tr>
<th>Variables</th>
<th>L</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>H</th>
<th>H-L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel A: Simple Emissions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R]-R_f$ (%)</td>
<td>7.31</td>
<td>7.82</td>
<td>7.99</td>
<td>8.46</td>
<td>12.84</td>
<td>5.52</td>
</tr>
<tr>
<td>[t]</td>
<td>2.46</td>
<td>2.29</td>
<td>2.72</td>
<td>2.51</td>
<td>4.13</td>
<td>3.18</td>
</tr>
<tr>
<td>Std (%)</td>
<td>14.17</td>
<td>15.97</td>
<td>13.55</td>
<td>15.29</td>
<td>14.48</td>
<td>9.73</td>
</tr>
<tr>
<td>SR</td>
<td>0.52</td>
<td>0.49</td>
<td>0.59</td>
<td>0.55</td>
<td>0.89</td>
<td>0.57</td>
</tr>
<tr>
<td>Panel B: Toxicity-adjusted Emissions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[R]-R_f$ (%)</td>
<td>6.90</td>
<td>8.31</td>
<td>7.99</td>
<td>7.89</td>
<td>12.77</td>
<td>5.87</td>
</tr>
<tr>
<td>[t]</td>
<td>2.34</td>
<td>2.44</td>
<td>2.73</td>
<td>2.43</td>
<td>4.10</td>
<td>3.24</td>
</tr>
<tr>
<td>Std (%)</td>
<td>14.16</td>
<td>15.97</td>
<td>13.71</td>
<td>15.01</td>
<td>14.49</td>
<td>9.23</td>
</tr>
<tr>
<td>SR</td>
<td>0.49</td>
<td>0.52</td>
<td>0.58</td>
<td>0.53</td>
<td>0.88</td>
<td>0.64</td>
</tr>
</tbody>
</table>
Table 6. Firm Characteristics

This table reports summary statistics for five simple emissions portfolios (Panel A) and five toxicity-adjusted emissions portfolios (Panel B). Simple emissions (Emissions) are measured as the simple summations of chemical pollutants over counties, and then divided by the book value of equity at the end of fiscal year t-1 at the firm-year level. Toxicity-adjusted emissions (Emissions_adj) are measured as the hazardous-weighted summation of chemical pollutants over counties, and then divided by book value of equity at the end of fiscal year t-1 at firm-year level. Variables of portfolio characteristics are described in Table 4. The sample period is July 1992-Dec 2015.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simple Emissions</th>
<th>Toxicity-adjusted Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>2</td>
</tr>
<tr>
<td>Emissions</td>
<td>0.06</td>
<td>0.43</td>
</tr>
<tr>
<td>Log ME</td>
<td>10.99</td>
<td>11.58</td>
</tr>
<tr>
<td>B/M</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>I/A</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>AG</td>
<td>1.13</td>
<td>1.11</td>
</tr>
<tr>
<td>ROE</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>R&amp;D/AT</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>O/AT</td>
<td>0.56</td>
<td>0.50</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Numbers</td>
<td>112</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 7: Predicative Regressions - Future Profitability

This table reports panel regressions of future profitability on their emissions, perceived probability of shocks, and their interactions, together with other firm characteristics. The sample period is from 1992 to 2014 and excludes financial industries. We control for industry fixed effects based on Fama-French 48 industry classifications. We measure the perceived possibility of policy regime shift shocks ("Shocks") using the log difference (i.e., the growth rate) of the total number of firms that report their toxic emissions ("Disclosure”), temperatures, and rainfall. All independent variables are normalized to a zero mean and a one standard deviation after winsorization at the 1th and 99th percentile of their empirical distribution. t-statistics clustered by firms with ***, **, * indicate statistical significance at the 1, 5, and 10% levels.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Disclosure</th>
<th>Temperature</th>
<th>Rainfall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions</td>
<td>-0.11***</td>
<td>0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>[t]</td>
<td>-5.53</td>
<td>0.59</td>
<td>-0.91</td>
</tr>
<tr>
<td>Shocks</td>
<td>-0.00</td>
<td>0.02</td>
<td>0.05***</td>
</tr>
<tr>
<td>[t]</td>
<td>-0.06</td>
<td>1.37</td>
<td>3.78</td>
</tr>
<tr>
<td>Emissions x Shocks</td>
<td>-0.12***</td>
<td>-0.10***</td>
<td>-0.03***</td>
</tr>
<tr>
<td>[t]</td>
<td>-6.25</td>
<td>-5.59</td>
<td>-2.68</td>
</tr>
<tr>
<td>Log ME</td>
<td>0.17***</td>
<td>0.17***</td>
<td>0.17***</td>
</tr>
<tr>
<td>[t]</td>
<td>10.34</td>
<td>10.39</td>
<td>10.38</td>
</tr>
<tr>
<td>Log B/M</td>
<td>-0.06***</td>
<td>-0.05***</td>
<td>-0.05***</td>
</tr>
<tr>
<td>[t]</td>
<td>-3.44</td>
<td>-2.86</td>
<td>-2.83</td>
</tr>
<tr>
<td>I/K</td>
<td>-0.04***</td>
<td>-0.04***</td>
<td>-0.04***</td>
</tr>
<tr>
<td>[t]</td>
<td>-2.84</td>
<td>-2.97</td>
<td>-2.71</td>
</tr>
<tr>
<td>AG</td>
<td>-0.05***</td>
<td>-0.04***</td>
<td>-0.04***</td>
</tr>
<tr>
<td>[t]</td>
<td>-3.35</td>
<td>-3.18</td>
<td>-3.19</td>
</tr>
<tr>
<td>ROE</td>
<td>0.13***</td>
<td>0.12***</td>
<td>0.12***</td>
</tr>
<tr>
<td>[t]</td>
<td>8.50</td>
<td>8.04</td>
<td>8.05</td>
</tr>
<tr>
<td>R&amp;D / AT</td>
<td>-0.05***</td>
<td>-0.04***</td>
<td>-0.04***</td>
</tr>
<tr>
<td>[t]</td>
<td>-3.19</td>
<td>-2.69</td>
<td>-2.72</td>
</tr>
<tr>
<td>O / AT</td>
<td>0.18***</td>
<td>0.17***</td>
<td>0.17***</td>
</tr>
<tr>
<td>[t]</td>
<td>10.71</td>
<td>10.17</td>
<td>10.14</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.04***</td>
<td>0.04***</td>
<td>0.04***</td>
</tr>
<tr>
<td>[t]</td>
<td>3.04</td>
<td>2.83</td>
<td>2.99</td>
</tr>
<tr>
<td>Observations</td>
<td>6,845</td>
<td>6,762</td>
<td>6,762</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 8: Predicative Regressions - Future Litigations

This table reports the predictive relation between future litigations and simple (toxicity-adjusted) emissions. The sample period is from 2003 to 2014 and excludes financial industries. We report coefficients estimated from Probit, Poisson count, and negative binomial regression. We also control for time fixed effects and industry fixed effects based on Fama-French 48 industry classifications. All independent variables are normalized to a zero mean and one standard deviation after winsorization at the 1st and 99th percentile of their empirical distribution. t-statistics based on standard errors are clustered by firms, and ***, **, * indicate statistical significance at the 1, 5, and 10% levels.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simple Emissions</th>
<th>Toxicity-adjusted Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probit</td>
<td>NB</td>
</tr>
<tr>
<td>Emissions</td>
<td>0.19***</td>
<td>0.51***</td>
</tr>
<tr>
<td>[t]</td>
<td>3.92</td>
<td>2.89</td>
</tr>
<tr>
<td>Log ME</td>
<td>0.80***</td>
<td>2.56***</td>
</tr>
<tr>
<td>[t]</td>
<td>9.03</td>
<td>10.97</td>
</tr>
<tr>
<td>Log B/M</td>
<td>0.15**</td>
<td>0.46**</td>
</tr>
<tr>
<td>[t]</td>
<td>2.30</td>
<td>2.31</td>
</tr>
<tr>
<td>I/K</td>
<td>-0.08</td>
<td>-0.42*</td>
</tr>
<tr>
<td>[t]</td>
<td>-1.28</td>
<td>-1.76</td>
</tr>
<tr>
<td>AG</td>
<td>-0.12**</td>
<td>-0.22*</td>
</tr>
<tr>
<td>[t]</td>
<td>-2.21</td>
<td>-1.83</td>
</tr>
<tr>
<td>ROE</td>
<td>0.01</td>
<td>0.37*</td>
</tr>
<tr>
<td>[t]</td>
<td>0.24</td>
<td>1.91</td>
</tr>
<tr>
<td>R&amp;D/AT</td>
<td>-0.00</td>
<td>-0.39</td>
</tr>
<tr>
<td>[t]</td>
<td>-0.00</td>
<td>-0.92</td>
</tr>
<tr>
<td>OC/AT</td>
<td>-0.01</td>
<td>-0.16</td>
</tr>
<tr>
<td>[t]</td>
<td>-0.07</td>
<td>-0.54</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.08</td>
<td>-0.38</td>
</tr>
<tr>
<td>[t]</td>
<td>-1.19</td>
<td>-1.39</td>
</tr>
<tr>
<td>Observations</td>
<td>5,058</td>
<td>5,978</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 9: Return Sensitivities to Perceived Probability of Shocks

This table reports panel regressions of individual stock excess returns on their emissions and other firm characteristics. The sample period is 1993 to 2015 and excludes financial industries. For each month from January of year t to December of year t, we compound monthly excess returns and then regress compounded excess returns of individual stock on simple emissions, perceived probability of shocks, and their interaction, together with different sets of variables that are known by the end of December of year t-1; we also control for industry fixed effects based on Fama-French 48 industry classifications. We present the time-series average and heteroscedasticity-robust t-statistics of the slopes (i.e., coefficients) estimated from the annual cross-sectional regressions for different model specifications. All independent variables are normalized to a zero mean and a one standard deviation after winsorization at the 1th and 99th percentile of their empirical distribution. We measure the perceived possibility of policy regime shift shocks ("Shocks") using the log difference (i.e., the growth rate) of the total number of firms that report their toxic emissions ("Disclosure"), temperature, and rainfall. Returns are annualized. t-statistics are estimated using the Newey-West correction, and ***, **, * indicate statistical significance at the 1, 5, and 10% levels.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Disclosure</th>
<th>Temperature</th>
<th>Rainfalls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions</td>
<td>0.17</td>
<td>0.48***</td>
<td>0.38**</td>
</tr>
<tr>
<td>[t]</td>
<td>0.98</td>
<td>2.89</td>
<td>2.42</td>
</tr>
<tr>
<td>Shocks</td>
<td>-0.98***</td>
<td>-0.09</td>
<td>-0.27**</td>
</tr>
<tr>
<td>[t]</td>
<td>-7.71</td>
<td>-0.72</td>
<td>-2.16</td>
</tr>
<tr>
<td>Emissions x Shocks</td>
<td>-2.12**</td>
<td>-0.42**</td>
<td>-0.25*</td>
</tr>
<tr>
<td>[t]</td>
<td>-2.28</td>
<td>-2.47</td>
<td>-1.87</td>
</tr>
<tr>
<td>Log ME</td>
<td>0.04</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>[t]</td>
<td>0.26</td>
<td>0.17</td>
<td>0.29</td>
</tr>
<tr>
<td>Log B/M</td>
<td>1.25***</td>
<td>1.28***</td>
<td>1.30***</td>
</tr>
<tr>
<td>[t]</td>
<td>7.47</td>
<td>7.56</td>
<td>7.67</td>
</tr>
<tr>
<td>I/K</td>
<td>-0.09</td>
<td>-0.16</td>
<td>-0.16</td>
</tr>
<tr>
<td>[t]</td>
<td>-0.67</td>
<td>-1.12</td>
<td>-1.12</td>
</tr>
<tr>
<td>AG</td>
<td>-0.56***</td>
<td>-0.60***</td>
<td>-0.61***</td>
</tr>
<tr>
<td>[t]</td>
<td>-3.84</td>
<td>-4.04</td>
<td>-4.10</td>
</tr>
<tr>
<td>ROE</td>
<td>-0.20</td>
<td>-0.18</td>
<td>-0.17</td>
</tr>
<tr>
<td>[t]</td>
<td>-1.29</td>
<td>-1.23</td>
<td>-1.19</td>
</tr>
<tr>
<td>R&amp;D/AT</td>
<td>0.16</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>[t]</td>
<td>0.95</td>
<td>1.30</td>
<td>1.29</td>
</tr>
<tr>
<td>OC/AT</td>
<td>0.56***</td>
<td>0.55***</td>
<td>0.55***</td>
</tr>
<tr>
<td>[t]</td>
<td>3.22</td>
<td>3.15</td>
<td>3.13</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.46***</td>
<td>0.46***</td>
<td>0.46***</td>
</tr>
<tr>
<td>[t]</td>
<td>3.28</td>
<td>3.28</td>
<td>3.33</td>
</tr>
<tr>
<td>Observations</td>
<td>9,851</td>
<td>9,669</td>
<td>9,669</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

43
Table 10: Asset Pricing Factor Tests

This table shows asset pricing factor tests for five portfolios sorted on emissions relative to their industry peers, for which we use the Fama-French 48 industry classifications and rebalance portfolios at the end of every June. The results use monthly data, where the sample period is from July 1992 to December 2015 and excludes financial industries. In Panel A, we report the portfolio alphas and betas by the Fama-French five-factor model, including MKT, SMB, HML, RMW, and CMA factors. In Panel B, we report portfolio alphas and betas by the HXZ q-factor model, including MKT, SMB, I/A, and ROE factors. Data on the Fama-French five-factor model are from Kenneth French's website. Data on the I/A and ROE factor are provided by Kewei Hou, Chen Xue, and Lu Zhang. Standard errors are estimated using the Newey-West correction with ***, **, and * indicate statistical significance at the 1, 5, and 10% levels. We include t-statistics and annualize the portfolio alphas multiplying by 12. All portfolios returns correspond to value-weighted returns by firm market capitalization. The sample period is July 1992-Dec 2015.

| Variables | L | 2 | 3 | 4 | H | H-L | L | 2 | 3 | 4 | H | H-L |
|-----------|---|---|---|---|---|-----|---|---|---|---|---|---|-----|
| **Panel A: Fama-French Five-factor Model** |
| α_{FF5}   | -1.36 | 0.15 | 1.11 | -2.29 | 3.89** | 5.25*** | -1.54 | 0.48 | 0.42 | -1.97 | 3.75** | 5.30*** |
| [t]       | -0.88 | 0.10 | 0.76 | -1.45 | 2.27 | 3.11 | -0.98 | 0.31 | 0.32 | -1.36 | 3.21 | 3.09 |
| MKT_Rf    | 0.94*** | 1.00*** | 0.85*** | 1.06*** | 0.90*** | -0.04 | 0.94*** | 1.00*** | 0.88*** | 1.02*** | 0.90*** | -0.03 |
| [t]       | 25.01 | 32.60 | 27.55 | 25.96 | 21.45 | -0.71 | 25.65 | 29.93 | 28.26 | 30.62 | 22.08 | -0.63 |
| SMB       | -0.07 | -0.13** | -0.23*** | -0.09* | -0.02 | -0.09** | -0.09** | -0.12** | -0.23** | -0.11* | -0.08 | 0.01 |
| [t]       | -1.44 | -2.55 | -4.80 | -1.77 | -1.49 | -0.27 | -2.01 | -2.34 | -4.72 | -1.79 | -1.27 | 0.20 |
| HML       | -0.04 | 0.06 | -0.02 | -0.04 | 0.03 | 0.06 | -0.02 | 0.03 | -0.01 | -0.05 | 0.04 | 0.06 |
| [t]       | -0.66 | 1.09 | -0.25 | -0.86 | 0.34 | 0.82 | -0.36 | 0.49 | -0.21 | -1.02 | 0.46 | 0.75 |
| RMW       | 0.17* | 0.05 | 0.13** | 0.33*** | 0.24*** | 0.07 | 0.16** | 0.07 | 0.17*** | 0.23** | 0.24*** | 0.09 |
| [t]       | 1.95 | 0.75 | 2.12 | 3.33 | 3.20 | 0.81 | 2.16 | 0.83 | 3.06 | 2.51 | 3.31 | 1.15 |
| CMA       | 0.36*** | 0.02 | 0.17** | 0.55*** | 0.42*** | 0.06 | 0.31*** | 0.07 | 0.24*** | 0.50*** | 0.41*** | 0.10 |
| [t]       | 2.74 | 0.21 | 2.16 | 4.46 | 4.47 | 0.51 | 2.65 | 0.64 | 3.09 | 4.53 | 4.43 | 0.95 |

| **Panel B: HXZ q-factor Model** |
| α_{HXZ}   | -1.06 | 1.05 | 0.43 | -1.98 | 4.00** | 5.06*** | -1.31 | 1.54 | -0.36 | -1.66 | 3.84** | 5.15*** |
| [t]       | -0.70 | 0.69 | 0.31 | -1.27 | 2.22 | 3.06 | -0.82 | 0.99 | -0.30 | -1.10 | 2.15 | 3.10 |
| MKT_Rf    | 0.92*** | 0.98*** | 0.88*** | 1.03*** | 0.89*** | -0.04 | 0.92*** | 0.97*** | 0.91*** | 1.00*** | 0.89*** | -0.03 |
| [t]       | 24.21 | 29.05 | 22.70 | 27.41 | 19.82 | -0.61 | 24.86 | 27.17 | 24.82 | 28.82 | 20.03 | -0.59 |
| SMB       | -0.07 | -0.18** | -0.22*** | -0.09* | -0.02 | -0.10** | -0.17*** | -0.21*** | -0.10 | -0.09 | 0.01 |
| [t]       | -1.59 | -4.09 | -4.50 | -1.77 | -1.83 | -0.38 | -2.24 | -3.71 | -4.53 | -1.43 | -1.58 | 0.19 |
| I/A       | 0.33*** | 0.04 | 0.19*** | 0.48*** | 0.46*** | 0.13 | 0.31*** | 0.04 | 0.27*** | 0.42*** | 0.46*** | 0.14 |
| [t]       | 2.94 | 0.57 | 2.70 | 4.38 | 4.71 | 1.29 | 3.00 | 0.54 | 3.57 | 3.94 | 4.70 | 1.61 |
| ROE       | 0.13* | -0.00 | 0.22*** | 0.27*** | 0.20*** | 0.07 | 0.12* | 0.00 | 0.27*** | 0.19*** | 0.20*** | 0.08 |
| [t]       | 1.73 | -0.02 | 4.04 | 3.26 | 2.69 | 0.79 | 1.78 | 0.05 | 5.60 | 2.60 | 2.82 | 1.05 |
Table 11: Fama-Macbeth Regressions

This table reports Fama-Macbeth regressions of individual stock excess returns on their emissions and other firm characteristics. The sample period is July 1992 to December 2015 and excludes financial industries. For each month from July of year t to June of year t+1, we regress monthly excess returns of individual stock on simple emissions and toxicity-adjusted emissions, respectively, with different sets of variables that are known by the end of June of year t; we also control for industry fixed effects based on Fama-French 48 industry classifications. We present the time-series average and heteroscedasticity-robust t-statistics of the slopes (i.e., coefficients) estimated from the monthly cross-sectional regressions for different model specifications. All independent variables are normalized to a zero mean and a one standard deviation after winsorization at the 1th and 99th percentile of their empirical distribution. We include t-statistics and annualize individual stock excess returns by multiplying 12. Standard errors are estimated using the Newey-West correction, and ***,**,* indicate statistical significance at the 1, 5, and 10% levels.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Simple Emissions</th>
<th>Toxicity-adjusted Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions</td>
<td>7.18**</td>
<td>9.85***</td>
</tr>
<tr>
<td></td>
<td>[t] 2.55</td>
<td>2.65</td>
</tr>
<tr>
<td>Log ME</td>
<td>-3.60***</td>
<td>-3.56***</td>
</tr>
<tr>
<td></td>
<td>[t] -4.88</td>
<td>-4.75</td>
</tr>
<tr>
<td>Log B/M</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>[t] 0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>I/A</td>
<td>-1.33*</td>
<td>-1.33*</td>
</tr>
<tr>
<td></td>
<td>[t] -1.96</td>
<td>-1.96</td>
</tr>
<tr>
<td>AG</td>
<td>2.12</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>[t] 0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>ROE</td>
<td>0.34</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>[t] 0.48</td>
<td>0.56</td>
</tr>
<tr>
<td>R&amp;D/AT</td>
<td>3.21***</td>
<td>3.22***</td>
</tr>
<tr>
<td></td>
<td>[t] 3.42</td>
<td>3.43</td>
</tr>
<tr>
<td>OC/AT</td>
<td>0.38</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>[t] 0.74</td>
<td>0.72</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.45</td>
<td>-0.43</td>
</tr>
<tr>
<td></td>
<td>[t] -0.44</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

Industry FE   Yes       Yes       Yes       Yes
Table 12. Simple Emissions across Fama-French 48 Industries

This table reports summary statistics of the firm-year observations of non-missing emissions (per millions of pounds) across industries, including the pooled mean (Mean), standard deviation (Std), minimum (Min), 25th percentile (Perc25), median (Perc50), 75th percentile (Perc75), and maximum (Max). The emissions are measured as the simple summations of chemical pollutants over counties at the firm-year level. Obs denotes the average number of firms with non-missing emissions in each industry. Industries are based on Fama-French 48 industry classifications (FF48), excluding financial industries. The sample period is 1991-2014.

<table>
<thead>
<tr>
<th>FF48</th>
<th>Industry Name</th>
<th>Obs</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
<th>Perc25</th>
<th>Medium</th>
<th>Perc75</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture</td>
<td>34</td>
<td>0.292</td>
<td>0.476</td>
<td>0.003</td>
<td>0.015</td>
<td>0.176</td>
<td>0.286</td>
<td>2.611</td>
</tr>
<tr>
<td>2</td>
<td>Food</td>
<td>686</td>
<td>2.800</td>
<td>14.574</td>
<td>0</td>
<td>0.004</td>
<td>0.142</td>
<td>0.892</td>
<td>250.298</td>
</tr>
<tr>
<td>3</td>
<td>Soda</td>
<td>118</td>
<td>0.353</td>
<td>0.566</td>
<td>0</td>
<td>0</td>
<td>0.133</td>
<td>0.258</td>
<td>2.446</td>
</tr>
<tr>
<td>4</td>
<td>Beer</td>
<td>120</td>
<td>1.068</td>
<td>1.822</td>
<td>0</td>
<td>0.064</td>
<td>1.049</td>
<td>6.078</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Tobacco</td>
<td>88</td>
<td>1.430</td>
<td>2.049</td>
<td>0</td>
<td>0.338</td>
<td>2.153</td>
<td>8.534</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Recreation</td>
<td>153</td>
<td>0.204</td>
<td>0.605</td>
<td>0</td>
<td>0.013</td>
<td>0.082</td>
<td>3.672</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Books</td>
<td>82</td>
<td>0.403</td>
<td>0.671</td>
<td>0</td>
<td>0</td>
<td>0.200</td>
<td>0.799</td>
<td>2.301</td>
</tr>
<tr>
<td>9</td>
<td>Household</td>
<td>790</td>
<td>8.536</td>
<td>25.299</td>
<td>0</td>
<td>0.529</td>
<td>5.182</td>
<td>261.204</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Apparel</td>
<td>86</td>
<td>0.197</td>
<td>0.301</td>
<td>0</td>
<td>0.092</td>
<td>0.221</td>
<td>1.438</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Healthcare</td>
<td>36</td>
<td>0.145</td>
<td>0.122</td>
<td>0</td>
<td>0.16</td>
<td>0.252</td>
<td>0.312</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Medical Equipment</td>
<td>526</td>
<td>0.539</td>
<td>1.286</td>
<td>0</td>
<td>0.069</td>
<td>0.361</td>
<td>6.775</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Drugs</td>
<td>493</td>
<td>7.273</td>
<td>18.983</td>
<td>0</td>
<td>0.131</td>
<td>3.327</td>
<td>122.074</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Chemicals</td>
<td>1290</td>
<td>38.941</td>
<td>176.595</td>
<td>0</td>
<td>1.311</td>
<td>11.221</td>
<td>443.010</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Rubber&amp;Plastic Products</td>
<td>365</td>
<td>1.043</td>
<td>1.970</td>
<td>0</td>
<td>0.109</td>
<td>1.147</td>
<td>15.129</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Textiles</td>
<td>256</td>
<td>0.370</td>
<td>0.625</td>
<td>0</td>
<td>0.044</td>
<td>0.571</td>
<td>4.774</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Construction Materials</td>
<td>1270</td>
<td>7.613</td>
<td>23.917</td>
<td>0</td>
<td>0.295</td>
<td>3.139</td>
<td>195.889</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Construction</td>
<td>248</td>
<td>2.463</td>
<td>14.850</td>
<td>0</td>
<td>0.160</td>
<td>0.706</td>
<td>142.007</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Steel</td>
<td>1091</td>
<td>13.285</td>
<td>28.648</td>
<td>0</td>
<td>0.886</td>
<td>9.048</td>
<td>281.487</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Fabricated Products</td>
<td>200</td>
<td>0.732</td>
<td>1.215</td>
<td>0</td>
<td>0.252</td>
<td>0.711</td>
<td>6.639</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Machinery</td>
<td>1427</td>
<td>1.814</td>
<td>6.017</td>
<td>0</td>
<td>0.227</td>
<td>1.374</td>
<td>95.231</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Electrical Equipment</td>
<td>644</td>
<td>17.591</td>
<td>55.976</td>
<td>0</td>
<td>0.113</td>
<td>2.396</td>
<td>341.146</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Automobiles</td>
<td>987</td>
<td>2.463</td>
<td>7.199</td>
<td>0</td>
<td>0.240</td>
<td>1.393</td>
<td>78.680</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Aircraft</td>
<td>354</td>
<td>24.422</td>
<td>94.970</td>
<td>0</td>
<td>0.138</td>
<td>3.274</td>
<td>620.793</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Ships</td>
<td>140</td>
<td>1.126</td>
<td>1.288</td>
<td>0</td>
<td>0.565</td>
<td>1.865</td>
<td>5.544</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Defense</td>
<td>73</td>
<td>0.814</td>
<td>1.506</td>
<td>0</td>
<td>0.180</td>
<td>0.497</td>
<td>5.634</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Precious Metals</td>
<td>66</td>
<td>6.436</td>
<td>18.298</td>
<td>0</td>
<td>0.032</td>
<td>0.500</td>
<td>81.536</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Mines</td>
<td>182</td>
<td>20.996</td>
<td>88.567</td>
<td>0</td>
<td>0.102</td>
<td>1.227</td>
<td>719.222</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Coal</td>
<td>99</td>
<td>0.072</td>
<td>0.191</td>
<td>0</td>
<td>0</td>
<td>0.021</td>
<td>1.155</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Oil</td>
<td>635</td>
<td>37.615</td>
<td>86.372</td>
<td>0</td>
<td>0.177</td>
<td>3.127</td>
<td>313.612</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Utilities</td>
<td>975</td>
<td>20.170</td>
<td>45.063</td>
<td>0</td>
<td>0.297</td>
<td>15.237</td>
<td>413.651</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Communication</td>
<td>106</td>
<td>2.792</td>
<td>6.486</td>
<td>0</td>
<td>0.120</td>
<td>1.010</td>
<td>33.286</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>Business Services</td>
<td>414</td>
<td>14.892</td>
<td>51.695</td>
<td>0</td>
<td>0.076</td>
<td>0.849</td>
<td>294.351</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>Computers</td>
<td>243</td>
<td>0.574</td>
<td>0.986</td>
<td>0</td>
<td>0.096</td>
<td>0.683</td>
<td>5.749</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Chips</td>
<td>1463</td>
<td>2.353</td>
<td>14.246</td>
<td>0</td>
<td>0.139</td>
<td>0.797</td>
<td>221.451</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Measuring&amp;Control Equipment</td>
<td>372</td>
<td>3.694</td>
<td>13.255</td>
<td>0</td>
<td>0.086</td>
<td>0.620</td>
<td>89.921</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Business Supplies</td>
<td>770</td>
<td>21.326</td>
<td>56.912</td>
<td>0</td>
<td>0.649</td>
<td>19.308</td>
<td>515.35</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Shipping Containers</td>
<td>321</td>
<td>13.46</td>
<td>30.205</td>
<td>0</td>
<td>0.806</td>
<td>6.411</td>
<td>145.161</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Transportation</td>
<td>169</td>
<td>2.254</td>
<td>5.392</td>
<td>0</td>
<td>0.177</td>
<td>0.591</td>
<td>22.869</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Wholesale</td>
<td>653</td>
<td>3.299</td>
<td>17.155</td>
<td>0</td>
<td>0.048</td>
<td>1.206</td>
<td>261.351</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Retail</td>
<td>294</td>
<td>2.303</td>
<td>8.278</td>
<td>0</td>
<td>0.044</td>
<td>0.300</td>
<td>76.887</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>Meals</td>
<td>85</td>
<td>7.257</td>
<td>16.357</td>
<td>0</td>
<td>0.032</td>
<td>11.31</td>
<td>79.198</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>Other</td>
<td>130</td>
<td>33.379</td>
<td>69.766</td>
<td>0</td>
<td>0.112</td>
<td>1.946</td>
<td>10.224</td>
<td>347.037</td>
</tr>
</tbody>
</table>
Table 13. Transition Matrix: Persistence of Simple and Toxicity-adjusted Emissions

This table presents transition frequency (%) across simple emissions quintiles in Panel A (and, respectively, toxicity-adjusted emissions quintiles in Panel B) from year $t$ to $t+1$ (column 1 to column 6) and from year $t$ to $t+5$ (column 7 to column 12). Simple emissions are measured as the simple summations of chemical pollutants of a firm in year $t-1$, and then divided by the book value of equity at the end of fiscal year $t-1$. Toxicity-adjusted emissions are measured as the hazardous-weighted summation of chemical pollutants of a firm in year $t-1$, and then divided by the book value of equity at the end of fiscal year $t-1$ at the firm-year level. The sample period is July 1992-Dec 2015.

### Panel A: Transition across Quintiles of Simple Emissions from year $t$ to year $t+1$ from year $t$ to year $t+5$

<table>
<thead>
<tr>
<th></th>
<th>L(t+1)</th>
<th>2(t+1)</th>
<th>3(t+1)</th>
<th>4(t+1)</th>
<th>H(t+1)</th>
<th>L(t+5)</th>
<th>2(t+5)</th>
<th>3(t+5)</th>
<th>4(t+5)</th>
<th>H(t+5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(t)</td>
<td>86.32</td>
<td>12.10</td>
<td>1.03</td>
<td>0.50</td>
<td>0.05</td>
<td>L(t)</td>
<td>70.79</td>
<td>22.07</td>
<td>4.24</td>
<td>1.95</td>
</tr>
<tr>
<td>2(t)</td>
<td>14.70</td>
<td>72.16</td>
<td>11.66</td>
<td>1.23</td>
<td>0.25</td>
<td>2(t)</td>
<td>26.30</td>
<td>52.02</td>
<td>16.00</td>
<td>4.90</td>
</tr>
<tr>
<td>3(t)</td>
<td>1.71</td>
<td>15.26</td>
<td>70.18</td>
<td>12.04</td>
<td>0.81</td>
<td>3(t)</td>
<td>9.15</td>
<td>26.64</td>
<td>45.87</td>
<td>15.78</td>
</tr>
<tr>
<td>4(t)</td>
<td>0.83</td>
<td>1.66</td>
<td>16.51</td>
<td>73.47</td>
<td>7.52</td>
<td>4(t)</td>
<td>3.14</td>
<td>6.99</td>
<td>28.99</td>
<td>49.56</td>
</tr>
<tr>
<td>H(t)</td>
<td>0.32</td>
<td>0.31</td>
<td>1.04</td>
<td>10.80</td>
<td>87.53</td>
<td>H(t)</td>
<td>1.68</td>
<td>2.15</td>
<td>5.59</td>
<td>20.44</td>
</tr>
</tbody>
</table>

### Panel B: Transition across Quintiles of Toxicity-adjusted Emissions from year $t$ to year $t+1$ from year $t$ to year $t+5$

<table>
<thead>
<tr>
<th></th>
<th>L(t+1)</th>
<th>2(t+1)</th>
<th>3(t+1)</th>
<th>4(t+1)</th>
<th>H(t+1)</th>
<th>L(t+5)</th>
<th>2(t+5)</th>
<th>3(t+5)</th>
<th>4(t+5)</th>
<th>H(t+5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(t)</td>
<td>85.87</td>
<td>12.35</td>
<td>1.30</td>
<td>0.43</td>
<td>0.05</td>
<td>L(t)</td>
<td>70.45</td>
<td>22.15</td>
<td>4.59</td>
<td>1.96</td>
</tr>
<tr>
<td>2(t)</td>
<td>15.25</td>
<td>70.96</td>
<td>12.39</td>
<td>1.21</td>
<td>0.20</td>
<td>2(t)</td>
<td>26.96</td>
<td>50.18</td>
<td>16.81</td>
<td>5.03</td>
</tr>
<tr>
<td>3(t)</td>
<td>1.68</td>
<td>16.13</td>
<td>68.59</td>
<td>12.72</td>
<td>0.88</td>
<td>3(t)</td>
<td>9.76</td>
<td>26.61</td>
<td>44.57</td>
<td>16.53</td>
</tr>
<tr>
<td>4(t)</td>
<td>0.83</td>
<td>1.90</td>
<td>16.71</td>
<td>71.65</td>
<td>8.90</td>
<td>4(t)</td>
<td>3.07</td>
<td>7.84</td>
<td>29.04</td>
<td>47.93</td>
</tr>
<tr>
<td>H(t)</td>
<td>0.34</td>
<td>0.27</td>
<td>1.14</td>
<td>12.01</td>
<td>86.25</td>
<td>H(t)</td>
<td>1.72</td>
<td>2.28</td>
<td>5.56</td>
<td>21.58</td>
</tr>
</tbody>
</table>
A Additional Empirical Evidence

In this section, we provide additional empirical evidence on the pollution premium.

A.1 Summary Statistics across Industries

In Table 12, we report the summary statistics of the simple emissions of firms in each industry according to the Fama and French (1997) 48 (FF48) industry classifications. Some industries have more firms reporting to the TRI database, such as the Chemicals industry and the Steel industry. There are comparatively large cross-industry variations in chemical emissions. Specifically, the standard deviation ranges from 176.595 for the Chemicals industry to 0.122 for the Health Care industry. Therefore, to make sure our results are not driven by any particular industry, we control for industry effects as detailed later.

[Place Table 12 about here]

A.2 Transition Matrix

Whether firms’ emission intensity is persistent or not is important for our analysis of the emission-return relation. To check the persistence, we sort firms by quintiles of emission measures each year and examine the transition across quintiles over time. We present this analysis in Table 13. The left side of Panel A shows the transition from year t to year t+1, while the right side shows the transition from year t to year t+5. Firms in the top or bottom quintiles of the distribution of simple emissions, the probability of staying in the same quintile in the next year (five years later) is above 85% (70%). Persistence is comparable when we consider toxicity-adjusted emissions in Panel B, where the probability of staying in the same quintile in the next year (five years later) is almost the same as reported in Panel A. The persistent emission intensity is intuitive because firms cannot easily adjust their production designs and processes. More importantly, such persistence has important asset pricing implications: if there is any emission-return relation, it should be attributed to long-lasting fundamental issues rather than transitory effects such as market sentiment or mispricing.

[Place Table 13 about here]
B Model Solution

Timeline
We consider an economy with a finite horizon \([0, T]\). Regime shifts occur at time \(\tau\), where \(\tau \in (0, T)\), and \(\tau^+\) denotes the timing of right after regime shifts.

Proof of Lemma 1
From the capital growth equation \(dB^i_t = B^i_t d\Pi^i_t\), where the stochastic process of \(d\Pi^i_t\) is given by equation (1), we obtain the following expression for firm \(i\)'s capital at time \(T\):

\[
B^i_T = B^i_\tau e^{\left(\mu + \xi i g - \frac{1}{2} \sigma^2 - \frac{1}{2} \sigma^2_1\right)(T-\tau) + \sigma (Z_T - Z_\tau) + \sigma_1 (Z^i_T - Z^i_\tau)}, \tag{B1}
\]

where \(g \equiv g^W\) when there is a weak regulation change and \(g \equiv g^S\) is there is a strong regulation change. Aggregating across firms, we obtain

\[
B_T = \int_0^1 B^i_T di = e^{\left(\mu - \frac{1}{2} \sigma^2 - \frac{1}{2} \sigma^2_1\right)(T-\tau) + \sigma (Z_T - Z_\tau)} \int_0^1 B^i_\tau e^{\xi^i g(T-\tau) + \sigma_1 (Z^i_T - Z^i_\tau)} di. \tag{B2}
\]

The Law of large numbers implies that

\[
\int_0^1 B^i_\tau e^{\xi^i g(T-\tau) + \sigma_1 (Z^i_T - Z^i_\tau)} di \rightarrow E^i \left[ B^i_\tau e^{g(T-\tau) + \sigma_1 (Z^i_T - Z^i_\tau)} \right]
= e^{g(T-\tau)} E^i \left[ B^i_\tau \right] E^i \left[ e^{\sigma_1 (Z^i_T - Z^i_\tau)} \right]
= B_\tau e^{g(T-\tau) + \frac{1}{2} \sigma^2_1(T-\tau)}, \tag{B3}
\]

where \(E^i\) is the operator of cross-sectional expectation. The second equality of equation (B3) presents the independence of \(B^i_\tau\) and \(Z^i_T - Z^i_\tau\). In the last step, the cross-sectional expectation of \(B^i_\tau\) denotes

\[
E^i \left[ B^i_\tau \right] = \int_0^1 B^i_\tau di = B_\tau, \tag{B4}
\]

and the expectation of \(E^i \left[ e^{\sigma_1 (Z^i_T - Z^i_\tau)} \right]\) implies the mean of lognormal distribution.

Proof of Proposition 1
Using the market clearing condition \(W_T = B_T\), we can use equation (11) to compute the expected utility at time \(T\) conditional on a strict or weak regulation. The expectation is conditional on the government’s information set, which includes the realization of the
environmental cost.

\[
E_\tau \left[ \frac{W_T^{1-\gamma}}{1-\gamma} \right] = \frac{B_1^{1-\gamma} e^{(1-\gamma)(\mu + g S - \frac{1}{2} \sigma^2) (T-\tau)} + \frac{1}{2} (1-\gamma)^2 \sigma^2 (T-\tau)}{1-\gamma} \quad (B5)
\]

\[
E_\tau \left[ \frac{W_T^{1-\gamma}}{1-\gamma} \right] = \Phi(C) B_1^{1-\gamma} e^{(1-\gamma)(\mu + g W - \frac{1}{2} \sigma^2) (T-\tau)} + \frac{1}{2} (1-\gamma)^2 \sigma^2 (T-\tau). \quad (B6)
\]

The claim of the proposition follows immediately from the optimality condition

\[
E_\tau \left[ \frac{W_T^{1-\gamma}}{1-\gamma} \left| S \right. \right] > E_\tau \left[ \frac{\Phi(C) W_T^{1-\gamma}}{1-\gamma} \right]. \quad (B7)
\]

Therefore,

\[
\frac{B_1^{1-\gamma} e^{(1-\gamma)(\mu + g S - \frac{1}{2} \sigma^2) (T-\tau)} + \frac{1}{2} (1-\gamma)^2 \sigma^2 (T-\tau)}{1-\gamma} > \Phi(C) B_1^{1-\gamma} e^{(1-\gamma)(\mu + g W - \frac{1}{2} \sigma^2) (T-\tau)} + \frac{1}{2} (1-\gamma)^2 \sigma^2 (T-\tau). \quad (B8)
\]

We specify the functional form of \( \Phi(C) \) as \( 1 + C \), and further rearrange the inequality above to obtain

\[
e^{(1-\gamma)g^S(T-\tau)} < \Phi(C)e^{(1-\gamma)g^W(T-\tau)} = (1 + e^c)e^{(1-\gamma)g^W(T-\tau)} \quad (B9)
\]

\[
e^{(\gamma-1)(g^W-g^S)(T-\tau)} - 1 < e^c \quad \log \left\{ e^{(\gamma-1)(g^W-g^S)(T-\tau)} - 1 \right\} < c. \quad (B10)
\]

The threshold for policy regime shifts denotes

\[
\zeta(\tau) \equiv \log \left\{ e^{(\gamma-1)(g^W-g^S)(T-\tau)} - 1 \right\}. \quad (B11)
\]

**Proof of Corollary 1**

We define \( n(c; a, b) \) as the p.d.f of a normal distribution with mean \( a \) and variance \( b \). The p.d.f conditional on information at time \( t \) is given by

\[
n(c; \hat{c}_t, \hat{\sigma}_t^2) = \int_{-\infty}^{\infty} n(c; \hat{c}_t, \hat{\sigma}_t^2) n(\hat{c}_t; \hat{c}_t, \hat{\sigma}_t^2 - \hat{\sigma}_t^2) d\hat{c}_t. \quad (B12)
\]
This follows from general properties of the normal distribution. The proof is to note that

\[
\begin{align*}
c &= c - \hat{c}_r + \hat{c}_r, \\
c - \hat{c}_r &\sim \text{Normal}(0, \hat{\sigma}_r^2), \\
\hat{c}_r &\sim \text{Normal}(\hat{c}_t, \hat{\sigma}_t^2 - \hat{\sigma}_r^2),
\end{align*}
\]

(B13)  (B14)  (B15)

where \(\hat{c}_r\) follows a normal distribution conditional on information at time \(t\). According to the dynamics of posterior mean in equation (8), the recursive expression is given by

\[
\hat{c}_r = \hat{c}_t + \int_t^\tau \hat{\sigma}_s^2 \eta^{-1} dZ_s^c.
\]

(B16)

Therefore, the conditional expectation based on information at time \(t\) denotes

\[
E_t[\hat{c}_r] = \hat{c}_t.
\]

(B17)

The variance denotes

\[
E_t[(\hat{c}_r - \hat{c}_t)^2] = \int_t^\tau \left( \hat{\sigma}_s^2 \eta^{-1} \right)^2 ds
\]

\[
= \frac{1}{\hat{\sigma}_t^2 + \frac{s}{\eta^2}} = \hat{\sigma}_t^2 - \hat{\sigma}_r^2.
\]

(B18)

Given the linearity of expectation operator,

\[
\begin{align*}
E_t[c] &= E_t[(c - \hat{c}_r) + \hat{c}_r] = E_t[(c - \hat{c}_r)] + E_t[\hat{c}_r] \\
&= E_t[E_r[(c - \hat{c}_r)] + E_t[\hat{c}_r] \\
&= 0 + \hat{c}_t \\
&= \hat{c}_t.
\end{align*}
\]

(B19)

We can also show that \(c - \hat{c}_r\) and \(\hat{c}_r\) are independent when two random variables are uncorrelated. The covariance is defined as

\[
\text{Cov}_t[(c - \hat{c}_r), \hat{c}_r] = E_t[(c - \hat{c}_r)\hat{c}_r] - E_t[(c - \hat{c}_r)]E_t[\hat{c}_r].
\]

(B20)
By using the law of iterated expectation, the first term in the RHS of equation (B20) denotes

\[
E_t[(c - \hat{c}_\tau)\hat{c}_\tau] = E_t[E_{\tau}[(c - \hat{c}_\tau)\hat{c}_\tau]] = E_t[E_{\tau}[(c - \hat{c}_\tau)\hat{c}_\tau]] = 0,
\]

and the second term shows zero. Therefore, we verify the independence implies \( \text{Cov}_t[(c - \hat{c}_\tau), \hat{c}_\tau] = 0 \). As a result, the variance based on information at time \( t \) denotes

\[
\text{Var}_t[c] = \text{Var}_t[(c - \hat{c}_\tau) + \hat{c}_\tau] = \text{Var}_t[c - \hat{c}_\tau] + \text{Var}_t[\hat{c}_\tau] + 2 \text{Cov}_t[(c - \hat{c}_\tau), \hat{c}_\tau] = \hat{\sigma}_t^2.
\]

Therefore, \( c \) follows a normal distribution condition on information at time \( t \)

\[
c \sim \text{Normal}(\hat{c}_t, \hat{\sigma}_t^2),
\]

and the probability of regime shifts at \( \tau \)

\[
p_{\tau|t} = 1 - N(c(\tau); \hat{c}_t, \hat{\sigma}_t^2).
\]

**Proof of Proposition 2**

Before the proof of Proposition 2, we need to prove the Lemma below.

**Lemma 2.** When policy regime shifts occur at time \( \tau \), the market value of each firm \( i \) takes one of two values

\[
M^i_{\tau+} = \begin{cases} 
M^{S,i}_{\tau+} = B^i_t e^{(\mu - \gamma \sigma^2 + \xi^i g^S)(T-\tau)} & \text{if regime shifts} \\
M^{W,i}_{\tau+} = B^i_t e^{(\mu - \gamma \sigma^2 + \xi^i g^W)(T-\tau)} & \text{if regime does not shift},
\end{cases}
\]

where \( \tau+ \) is the timing right after regime shifts. Unconditionally, firm \( i \)'s market value denotes

\[
M^i_\tau = E_\tau[M^i_{\tau+}] = p_\tau M^{S,i}_{\tau+} + (1 - p_\tau)M^{W,i}_{\tau+}.
\]

**Proof of Lemma 2**

The state price density is \( \pi_t = \frac{1}{\kappa}E_t[B^{-\gamma}_T] \). Its value, when regime shifts occur at time \( \tau \), is
given by

\[ \pi_{\tau+} = \kappa^{-1} B_{\tau}^{-\gamma} \pi_{\tau+} \left[ e^{-\gamma(\mu + g^{i} - \frac{1}{2} \sigma^{2})(T - \tau) - \gamma \sigma(Z_T - Z_{\tau})} \right] \]

\[ = \begin{cases} 
\kappa^{-1} B_{\tau}^{-\gamma} \pi_{\tau+} \left[ e^{-\gamma(\mu + g^{i} - \frac{1}{2} \sigma^{2})(T - \tau) - \gamma \sigma(Z_T - Z_{\tau})} \right] & \text{if regime shifts} \\
\kappa^{-1} B_{\tau}^{-\gamma} \pi_{\tau+} \left[ e^{-\gamma(\mu + g^{i} - \frac{1}{2} \sigma^{2})(T - \tau) - \gamma \sigma(Z_T - Z_{\tau})} \right] & \text{if regime does not shift}
\end{cases} \]

\[ \pi^{S}_{\tau+} = \kappa^{-1} B_{\tau}^{-\gamma} e^{\left(\frac{1}{2}(\gamma + 1)\sigma^{2}\right)T - \tau} \]

\[ \pi^{W}_{\tau+} = \kappa^{-1} B_{\tau}^{-\gamma} e^{\left(\frac{1}{2}(\gamma + 1)\sigma^{2}\right)T - \tau} \]

where we use the definition of equation (11). We can infer the state price density at time \( \tau \)

\[ \pi_{\tau} = E_{\tau}^{T}[\pi_{\tau+}] = p_{\tau} \pi^{S}_{\tau+} + (1 - p_{\tau}) \pi^{W}_{\tau+}, \]  

(B28)

where \( p_{\tau} \) is the probability of a policy change from the perspective of investor. The market value of stock \( i \) is given by

\[ M_{T}^{i} = E_{T}^{T} \left[ \frac{\pi_{T}}{\pi_{\tau+}} B_{T}^{i} \right]. \]  

(B29)

After policy regime shifts at time \( \tau \), using the results of equation (33), we obtain

\[ E_{\tau+}^{T} [\pi_{T} B_{T}^{i} | S] = \kappa^{-1} E_{\tau+}^{T} [B_{T}^{-\gamma} B_{T}^{i} | S] = \kappa^{-1} B_{\tau}^{-\gamma} B_{\tau}^{i} E_{\tau+}^{T} \left[ e^{(1 - \gamma)(\mu - \frac{1}{2} \sigma^{2})(T - \tau) + (\xi' - \gamma)g^{i}(T - \tau) + (1 - \gamma) \sigma(Z_T - Z_{\tau})} | S \right] \]

\[ \times E_{\tau+}^{T} \left[ e^{-\frac{1}{2} \sigma^{2}(T - \tau) + \gamma \sigma(Z_{T} - Z_{\tau})} \right] \]

\[ = \kappa^{-1} B_{\tau}^{-\gamma} B_{\tau}^{i} E_{\tau+}^{T} \left[ e^{(1 - \gamma)(\mu - \frac{1}{2} \sigma^{2})(T - \tau) + (\xi' - \gamma)g^{i}(T - \tau) + (1 - \gamma) \sigma(Z_T - Z_{\tau})} | S \right] \]

\[ = \kappa^{-1} B_{\tau}^{-\gamma} B_{\tau}^{i} E_{\tau+}^{T} \left[ e^{(1 - \gamma)(\mu - \frac{1}{2} \sigma^{2})(T - \tau) + (\xi' - \gamma)g^{i}(T - \tau) + (1 - \gamma) \sigma(Z_T - Z_{\tau})} \right] \]

\[ = \kappa^{-1} B_{\tau}^{-\gamma} B_{\tau}^{i} e^{(1 - \gamma)(\mu - \frac{1}{2} \sigma^{2})(T - \tau) + (\xi' - \gamma)g^{i}(T - \tau) + \frac{1}{2}(1 - \gamma)^{2} \sigma^{2}(T - \tau)} \]  

(B30)

\[ E_{\tau+}^{T} [\pi_{T} B_{T}^{i} | S] = \kappa^{-1} B_{\tau}^{-\gamma} B_{\tau}^{i} e^{(1 - \gamma)(\mu - \frac{1}{2} \sigma^{2})(T - \tau) + (\xi' - \gamma)g^{i}(T - \tau) + \frac{1}{2}(1 - \gamma)^{2} \sigma^{2}(T - \tau)}, \]  

(B31)

where the derivations of \( E_{\tau+}^{T} [\pi_{T} B_{T}^{i} | S] \) are analogous to those of \( E_{\tau+}^{T} [\pi_{T} B_{T}^{i} | S] \). We can obtain firm \( i \)'s stock price after policy regime shifts

\[ M_{\tau+}^{S,i} = E_{\tau+}^{T} \left[ \frac{\pi_{T}}{\pi_{\tau+}^{S}} B_{T}^{i} | S \right] = \frac{E_{\tau+}^{T} \left[ \pi_{T} B_{T}^{i} | S \right]}{E_{\tau+}^{T} \left[ \pi_{\tau+}^{S} \right]} = B_{\tau+}^{i} e^{(\mu - \gamma \sigma^{2} + \xi' g^{i})(T - \tau)} \]  

(B32)
and

\[ M_{r+}^{W,i} = E_{\pi^+} \left[ \frac{\pi_T}{\pi_{r+}} B_T^i \big| W \right] = \frac{E_{\pi^+}[\pi_T B_T^i \big| W]}{\pi_{r+}} = B_{r+}^i e^{(\mu - \gamma \sigma^2 + \xi^W)(T - \tau)}. \]  

(B33)

Finally, the stock price at time \( \tau \) when the policy regime change is equal to

\[ M_r^i = E_{\pi^+} \left[ \frac{\pi_T}{\pi_{r+}} B_T^i \right] = \frac{1}{\pi_{r+}} \left[ E_{\pi^+}[\kappa^{-1} B_T^{-\gamma} B_T^i] \right] \]

\[ = \frac{p_r E_{\pi^+}[\kappa^{-1} B_T^{-\gamma} B_T^i \big| S] + (1 - p_r) E_{\pi^+}[\kappa^{-1} B_T^{-\gamma} B_T^i \big| W]}{\pi_{r+}} \]

\[ = \frac{p_r \pi^S_{r+} M_{r+}^{S,i} + (1 - p_r) \pi^W_{r+} M_{r+}^{W,i}}{p_r \pi^S_{r+} + (1 - p_r) \pi^W_{r+}} \]

\[ = \phi_r M_{r+}^{S,i} + (1 - \phi_r) M_{r+}^{W,i}, \]  

(B34)

where

\[ \phi_r \equiv \frac{p_r \pi^S_{r+}}{p_r \pi^S_{r+} + (1 - p_r) \pi^W_{r+}} \]

\[ = \frac{p_r}{p_r + (1 - p_r) \pi^W_{r+}} \]

\[ = \frac{p_r}{p_r + (1 - p_r) e^{-\gamma(g^W - g^S)(T - \tau)}} \]  

(B35)

and

\[ G_r^i \equiv \frac{M_{r+}^{W,i}}{M_{r+}^{S,i}} = e^{\beta^i(g^W - g^S)(T - \tau)}. \]  

(B36)

**Proof of Proposition 2**

The state price density the expected value of whether environmental policy regime shifts or not,

\[ \pi_t = E_t[\pi_{r+}] \]

\[ = E_t[p_r \pi^S_{r+} + (1 - p_r) \pi^W_{r+}] \]

\[ = E_t[p_r E_t[\pi^S_{r+}] + E_t[(1 - p_r)] E_t[\pi^W_{r+}]] \]

\[ = p_r E_t[\pi^S_{r+}] + (1 - p_r) E_t[\pi^W_{r+}], \]  

(B37)

54
where

\[
\pi^S_t = E_t[\pi^S_{\tau+}],
\]

(B38)

\[
\pi^W_t = E_t[\pi^W_{\tau+}],
\]

(B39)

and \(p_{\tau|t}\) refers to Corollary 1. We can show that

\[
E_t[p_{\tau}] = E_t\left[ \int_{\mathbb{R}} \int_{\mathbb{R}} n(c; \hat{c}_\tau, \hat{\sigma}_\tau^2)dc \right]
\]

(B40)

Recalling that equation (B27) the state price density after the government decides whether to change its environmental regulation or not, its value conditional on time \(t \leq \tau\) is characterized as follows.

\[
\pi^S_t = E_t\left[ \pi^S_{\tau+} \right] = E_t\left[ \kappa^{-1} B_{\tau+}^{\gamma} \exp\left\{ -\gamma (\mu^S + g^S + \frac{1}{2} \gamma (\gamma+1) \sigma^2) \right\} (T-\tau) \right]
\]

(B41)

Recalling that equation (B27) the state price density after the government decides whether to change its environmental regulation or not, its value conditional on time \(t \leq \tau\) is characterized as follows.

\[
\pi^S_t = E_t\left[ \pi^S_{\tau+} \right] = E_t\left[ \kappa^{-1} B_{\tau+}^{\gamma} \exp\left\{ -\gamma (\mu^S + g^S + \frac{1}{2} \gamma (\gamma+1) \sigma^2) \right\} (T-\tau) \right]
\]

On the other hand, we can immediately obtain the state price density at time \(t\), given that the economy starts from the weak regulation, according to equation (1). We solve the expectation problem by substituting the recursive expression of \(B_{\tau}\) into the expectation.

\[
B_{\tau} = B_t e^{\mu (\tau-t) + g^W (\tau-t) - \frac{1}{2} \sigma^2 (\tau-t) + \sigma (Z_\tau - Z_t)}.
\]
there is no regulation regime change.

\[ \pi_t^W = E_t[\pi_{t+}^W] = B_t^{-\gamma}e \left\{ -\gamma(\mu_g^W+\frac{1}{2}\gamma(\gamma+1)\sigma^2) \right\}(T-t). \] (B42)

Finally, we obtain the state price of density at time \( t \) conditional on the government changes the regulation. The unconditional state price of density denotes

\[ \pi_t = p_{t\mid t}\pi_t^S + (1 - p_{t\mid t})\pi_t^W \]

\[ = p_{t\mid t}B_t^{-\gamma}e \left\{ -\gamma\mu_g^W + \frac{1}{2}\gamma(\gamma+1)\sigma^2 \right\}(T-t) - \gamma g^W(T-t) + (1 - p_{t\mid t})B_t^{-\gamma}e \left\{ -\gamma(\mu_g^W+\frac{1}{2}\gamma(\gamma+1)\sigma^2) \right\}(T-t) \]

\[ = B_t^{-\gamma}e \left\{ -\gamma\mu_g^W + \frac{1}{2}\gamma(\gamma+1)\sigma^2 \right\}(T-t) - \gamma g^W(T-t) \left[ p_{t\mid t}e^{-\gamma g^S(T-t)} + (1 - p_{t\mid t})e^{-\gamma g^W(T-t)} \right] \]

\[ = B_t^{-\gamma}\Omega_t. \] (B43)

where

\[ \Omega_t = e \left\{ -\gamma\mu_g^W + \frac{1}{2}\gamma(\gamma+1)\sigma^2 \right\}(T-t) - \gamma g^W(T-t) \left[ p_{t\mid t}e^{-\gamma g^S(T-t)} + (1 - p_{t\mid t})e^{-\gamma g^W(T-t)} \right]. \] (B44)

**Proof of Proposition 3**

The SDF dynamics stem from an application of Ito’s Lemma to equation (18).

\[ \frac{d\pi_t}{\pi_t} = E_t \left[ \frac{d\pi_t}{\pi_t} \right] - \lambda dZ_t + \lambda_{c,t}d\hat{Z}_t^c. \] (B45)

Trivially, the price of risk of fundamental shocks denotes

\[ \lambda = \gamma \sigma, \] (B46)

The price of risk of uncertainty shocks denotes

\[ \lambda_{c,t} = \frac{1}{\Omega_t} \frac{\partial \Omega_t}{\partial p_{t\mid t}} \frac{\partial p_{t\mid t}}{\partial \hat{c}_t} \sigma_{c,t}^2 \eta^{-1} \]

\[ = \frac{e \left\{ -\gamma\mu_g^W + \frac{1}{2}\gamma(\gamma+1)\sigma^2 \right\}(T-t) - \gamma g^W(T-t) \left[ p_{t\mid t}e^{-\gamma g^S(T-t)} + (1 - p_{t\mid t})e^{-\gamma g^W(T-t)} \right]}{e \left\{ -\gamma\mu_g^W + \frac{1}{2}\gamma(\gamma+1)\sigma^2 \right\}(T-t) - \gamma g^W(T-t) \left[ p_{t\mid t}e^{-\gamma g^S(T-t)} + (1 - p_{t\mid t})e^{-\gamma g^W(T-t)} \right] \times n(g; \hat{c}_t, \hat{\sigma}_{c,t}^2) \times \hat{\sigma}_{c,t}^2 \eta^{-1}} \]

\[ = \left[ \frac{(1 - p_{t\mid t})(1 - F_T)}{p_{t\mid t} + (1 - p_{t\mid t})F_T} \right] n(g; \hat{c}_t, \hat{\sigma}_{c,t}^2) \sigma_{c,t}^2 \eta^{-1}, \] (B47)
where

\[ F_\tau = \frac{e^{-\gamma T - \gamma \tau}}{e^{-\gamma S (T - \tau)}} = e^{-\gamma (gW - gS)(T - \tau)}. \]  

(B48)

**Proof of Proposition 4**

The proof is a continuation of the Proposition 3. For \( t < \tau \), market value satisfies

\[ M_t^i = E_t \left[ \frac{\pi_t}{\pi_t} M_T^i \right]. \]

Firm \( i \)'s stock price denotes

\[ M_{t,i}^{S} = \frac{E_t \left[ \pi_t^S + M_{r+}^{S,i} \right]}{\pi_t^S} = B_t e^{(\mu - \gamma \sigma^2)(T - t) + \xi^i gW (\tau - t) + \xi^i gS (T - \tau)}, \]  

(B49)

when regime shifts at time \( \tau \), and denotes

\[ M_{t,i}^{W} = \frac{E_t \left[ \pi_t^W + M_{r+}^{W,i} \right]}{\pi_t^W} = B_t e^{(\mu - \gamma \sigma^2 + \xi^i gW)(T - t)}, \]  

(B50)

57
when regime does not shift at time \( \tau \). Following Proposition 3, firm \( i \)'s stock price is determined by using law of iterated expectation.

\[
M^i_t = E_t \left[ \frac{\pi_T}{\pi_t} B_T^i \right] = \frac{1}{\pi_t} E_t \left[ E_{\tau} [\kappa^{-1} B_T^{-\gamma} B_T^i] \right]
\]

\[
= \frac{1}{\pi_t} E_t \left[ p_{\tau} E_{\tau+} [\kappa^{-1} B_T^{-\gamma} B_T^i \mid S] + (1 - p_{\tau}) E_{\tau+} [\kappa^{-1} B_T^{-\gamma} B_T^i \mid W] \right]
\]

\[
= \frac{\pi_t}{\pi_t} p_{\tau|t} E_t \left[ \kappa^{-1} B_T^{-\gamma} e \left( -\gamma (\mu + g^S) + \frac{1}{2} \gamma (\gamma + 1) \sigma^2 \right) (T - \tau) B_T^i e^{(\mu - \gamma \sigma^2 + \xi^W g^S) (T - \tau)} \right]
\]

\[
+ \frac{\pi_t}{\pi_t} (1 - p_{\tau|t}) E_t \left[ \kappa^{-1} B_T^{-\gamma} e \left( -\gamma (\mu + g^W) + \frac{1}{2} \gamma (\gamma + 1) \sigma^2 \right) (T - \tau) B_T^i e^{(\mu - \gamma \sigma^2 + \xi^W g^S) (T - \tau)} \right]
\]

\[
= \frac{p_{\tau|t} e^{-\gamma g^S (T - \tau)} M^S,i_t + (1 - p_{\tau|t}) e^{-\gamma g^S (T - \tau)} M^W,i_t}{p_{\tau|t} e^{-\gamma g^S (T - \tau)} + (1 - p_{\tau|t}) e^{-\gamma g^W (T - \tau)}}
\]

\[
= \frac{p_{\tau|t} e^{-\gamma g^S (T - \tau)} + (1 - p_{\tau|t}) e^{-\gamma g^W (T - \tau)}}{p_{\tau|t} + (1 - p_{\tau|t}) e^{-\gamma (g^W - g^S) (T - \tau)}}
\]

\[
= \phi_t M^S,i_t + (1 - \phi_t) M^W,i_t,
\]  

\[\text{(B51)}\]

where

\[
\phi_t \equiv \frac{p_{\tau|t}}{p_{\tau|t} + (1 - p_{\tau|t}) e^{-\gamma (g^W - g^S) (T - \tau)}}. \]  

\[\text{(B52)}\]

We can obtain firm \( i \)'s market valuation unconditionally by substituting equation (B49) and (B50) into the last equity in equation (B52)

\[
M^i_t = \phi_t M^S,i_t + (1 - \phi_t) M^W,i_t
\]

\[
= B_t e^{(\mu - \gamma \sigma^2) (T - \tau) + \xi^W (T - \tau)} \left[ \phi_t e^{\xi^S g^S (T - \tau)} + (1 - \phi_t) e^{\xi^W g^W (T - \tau)} \right]
\]

\[
= B_t \Theta^i_t,
\]  

\[\text{(B53)}\]
where
\[ \Theta_t^i = e^{(\mu - \gamma \sigma^2)(T-t) + \xi^i g^W(\tau-t)} [\phi_t e^{\xi^i g^S(T-\tau)} + (1 - \phi_t) e^{\xi^i g^W(T-\tau)}]. \]  

(B54)

**Proof of Proposition 5**

An application of Ito’s Lemma to equation (B26) characterizes the return dynamics as follows.

\[ \frac{dM^i_t}{M_t^i} = \mathbb{E}_t \left[ \frac{dM^i_t}{M_t^i} \right] + \sigma_i dZ_t + \sigma_i dZ_t^i + \beta^i_{M,t} d\hat{Z}_t^e, \]  

(B55)

where \( \sigma^i_{c,t} \) is the risk exposure to uncertainty shocks. The derivations of \( \sigma^i_{c,t} \) are as follows

\[ \beta^i_{M,t} = \frac{1}{\Theta_t^i} \frac{\partial \Theta_t^i}{\partial \phi_t} \frac{\partial \phi_t}{\partial \beta^i_{M,t}} \hat{\sigma}_{c,t} \hat{\sigma}_{c,t} \eta^{-1} \]

\[ = \frac{e^{(\mu - \gamma \sigma^2)(T-t) + \xi^i g^W(\tau-t)}}{e^{(\mu - \gamma \sigma^2)(T-t) + \xi^i g^W(\tau-t)}} \left[ \phi_t e^{\xi^i g^S(T-\tau)} + (1 - \phi_t) e^{\xi^i g^W(T-\tau)} \right] \times \]

\[ \left[ p_{r|t} + (1 - p_{r|t}) e^{-\gamma (g^W - g^S)(T-\tau)} \right] \left[ e^{-\gamma (g^W - g^S)(T-\tau)} \right] \]

\[ = \left[ \frac{1 - G^i_{\tau}}{\phi_t + (1 - \phi_t) G^i_{\tau}} \right] \left[ \frac{F_{\tau}}{(p_{r|t} + (1 - p_{r|t}) F_{\tau})^2} \right] n(\xi(\tau); \hat{c}_t, \hat{\sigma}^2_{c,t}) \hat{\sigma}^2_{c,t} \eta^{-1}. \]  

(B56)

**Proof of Corollary 2**

We present the partial derivative of \( \sigma^i_{c,t} \) to its dependence on \( \beta^i \).

\[ \frac{\partial \beta^i_{M,t}}{\partial \xi^i} = \frac{\partial}{\partial \xi^i} \left\{ \left[ \frac{1 - G^i_{\tau}}{\phi_t + (1 - \phi_t) G^i_{\tau}} \right] \left[ \frac{F_{\tau}}{(p_{r|t} + (1 - p_{r|t}) F_{\tau})^2} \right] n(\xi(\tau); \hat{c}_t, \hat{\sigma}^2_{c,t}) \hat{\sigma}^2_{c,t} \eta^{-1} \right\} \]

\[ = \left[ \frac{F_{\tau}}{(p_{r|t} + (1 - p_{r|t}) F_{\tau})^2} \right] n(\xi(\tau); \hat{c}_t, \hat{\sigma}^2_{c,t}) \hat{\sigma}^2_{c,t} \eta^{-1} \times \frac{\partial}{\partial \xi^i} \left\{ \left[ \frac{1 - G^i_{\tau}}{\phi_t + (1 - \phi_t) G^i_{\tau}} \right] \right\}. \]  

(B57)

Since only \( G^i_{\tau} \) depends on \( \xi^i \), our analysis focuses on terms related to \( G^i_{\tau} \).

\[ \frac{\partial}{\partial \xi^i} \left\{ \left[ \frac{1 - G^i_{\tau}}{\phi_t + (1 - \phi_t) G^i_{\tau}} \right] \right\} = \frac{\partial G^i_{\tau}}{\partial \xi^i} [\phi_t + (1 - \phi_t) G^i_{\tau}] - \frac{(1 - G^i_{\tau})}{\phi_t + (1 - \phi_t) G^i_{\tau}} < 0, \]  

(B58)
where $G^i_\tau > 1$ and $\partial G^i_\tau / \partial \xi^i > 0$. 