Where DoActivist Shareholders Trade?*

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Abstract
Before intervening in corporate governance, an activist needs to acquire a large ownership in a firm, and her information is private until she files Schedule 13D. We analyze how this large informed trader chooses between the lit exchange and the dark pool. We find that (1) the market share of the dark pool increases when an activist trades, (2) dark pool’s market share increases more when the activist’s value of information is higher, and (3) price discovery ahead of a Schedule 13D filing decreases with the activist’s value of information and the historical market share of the dark pool.

Keywords: Shareholder activism, Schedule 13D filing, Dark pool, Rational expectation equilibrium

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Over the past decades, shareholder activism has emerged as a new form of corporate governance mechanism (Brav et al., 2009, 2015a; Denes et al., 2017). To pursue their governance agenda, activists need to accumulate a large ownership in a target firm. Before an activist exceeds 5% ownership in a firm, which triggers the filing of Schedule 13D with the U.S. Securities and Exchange Commission (SEC), her intention to intervene is private information (Collin-Dufresne and Fos, 2015). As the only informed trader for the Schedule 13D event, the activist can profit from trading in more than 10 stock exchanges and 40 active dark pools, but the literature provides no information on where she trades. In this paper, we analyze, both theoretically and empirically, where a large informed trader like an activist trades before she reveals her intention.

To guide our empirical analysis, we start with a model with a lit exchange and a dark pool. The lit exchange is organized as in Kyle (1985), where a market maker sets the break-even price to clear the imbalance between the buy and sell orders. The model includes two types of traders: (1) an informed trader who knows the value of the asset at date 0, and (2) many liquidity traders who trade for exogenous reasons. On date 1, the exchange opens. The market maker observes the aggregate order flow of the informed and liquidity traders. He sets the price at the conditional expectation of the value of the asset. On date 2, the dark pool opens, and it uses the price set by the exchange to match buy and sell orders. Orders on the side with fewer shares get full execution while orders on the side with more shares get partial execution.

The main trade-off in this model is between two endogenous trading costs: the price impact in the lit exchange and the probability impact in the dark pool. In the exchange, each additional share from the informed trader moves the price towards a more unfavorable level. Orders submitted to the dark pool do not directly move the price, but the execution probability of an order decreases with the order size. We show that the informed trader has an incentive to trade in both
the lit exchange and the dark pool, because the price impact would be zero if the informed trader did not trade in the exchange and the execution probability would be very high if the informed trader only sends an infinitesimal order to the dark pool. An increase in liquidity trading in the exchange dilutes the price impact, giving the informed trader a stronger incentive to trade in the exchange. Similarly, an increase in liquidity trading in the dark pool reduces the probability impact, giving the informed trader a stronger incentive to trade in the dark pool.

The main mechanism driving our results is the externality of the exchange’s price impact on the dark pool. Compared to Kyle (1985), the informed trader in our model trades less aggressively on the exchange, because the price set by the exchange will be used to match orders in the dark pool. Such externality is stronger when the value of the private information is higher. The externality also increases with liquidity trading in the dark pool, because the informed trader can execute more shares in the dark pool. Intuitively, a higher information value gives the informed trader a stronger incentive to hide, while a more liquid dark pool creates more room for the informed trader to hide. Therefore, the market share of the dark pool increases with both the value of information and the volume of liquidity trading in the dark pool.

The informed traders’ order-splitting strategy in our model generates two different predictions on price discovery from Kyle (1985). In the one-period Kyle (1985) model, price reveals half of the information for any value of information and any level of liquidity trading. The addition of a dark pool destroys this invariance. First, the dark pool reduces the price discovery to less than half because of the externality of the exchange’s price impact on the dark pool. When the informed trader can match orders in the dark pool without directly moving the price, she trades less aggressively in the exchange and reveals less information. Second, price discovery now depends on the value of information and level of liquidity trading. An increase in the value of
information reduces price discovery, because the informed trader has a higher incentive to hide her information by trading more conservatively in the exchange. An increase in the liquid trading in the dark pool relative to the exchange also reduces price discovery, because the informed trader trades less aggressively in the exchange when the dark pool creates more room for her to hide information.

Trades from Schedule 13D filings provide a unique laboratory to test our model predictions because an activist is the unique informed trader before she releases her intention to intervene in corporate governance. The trades from Schedule 13D filings are both large and informed. In our sample period of 2014 – 2018, Schedule 13D filers acquire 4.78% of outstanding shares in the target firms in the 60 calendar days prior to the filing date, around 40% (= 1.86%/4.78%) of which are acquired over the event week when aggregate beneficial ownership exceeds the SEC’s 5% reporting threshold. An average Schedule 13D filing in our sample is characterized by a 4.05% cumulative abnormal return (CAR) in the \((t - 1, t + 11)\) trading-day window and 9.62% CAR in the \((t - 46, t + 15)\) trading-day window around the filing date.

Following the difference-in-differences approach in Collin-Dufresne and Fos (2015), we compare weeks with and without Schedule 13D trades to matched non-target firms with similar characteristics. We find that dark pool market share increases by 1.9% during weeks with Schedule 13D trades, representing a 13% increase relative to the mean dark pool market share in the weeks without Schedule 13D trades (14.57%). Next, we use the CAR of the target firm in the \((t - 46, t + 15)\) trading-day window around the filing date as the proxy for the value of the Schedule 13D filer’s private information. We find that a one standard deviation increase in the value of private information is associated with a 0.84% more increase in the market share of dark pools during weeks with informed trades. We also find a one standard deviation increase in the historical
market-adjusted dark pool market share of the target firm, a proxy for the relative liquidity trading in the dark pool, is associated with a 0.48% more increase in the market share of dark pools during weeks with informed trades.

We follow Weller (2017) to measure price discovery at the event level with a jump ratio, i.e., the ratio of the CAR over the \((t - 1, t + 15)\) trading-day window to that over \((t - 46, t + 15)\) trading-day window around the filing date. The average jump ratio in our sample of Schedule 13D events is 20.13%. A higher jump ratio indicates less price discovery in the pre-filing window. We find that a one standard deviation increase in the value of private information is associated with a 7.72% higher jump ratio, and a one standard deviation increase in the historical market-adjusted dark pool market share is associated with a 7.30% higher jump ratio. These results suggest that price discovery decreases with the value of private information, as well as the relative liquidity trading in the dark pool.

It has long been recognized in the corporate governance literature that activists need to acquire a large stake to voice their concerns about a firm, but does not consider where activists acquire these shares. The closest paper to ours is Collin-Dufresne and Fos (2016), who study how an informed trader times her trades when liquidity trading is stochastic but exogenous. Our paper complements their study by showing that activists not only choose when they trade but where they trade.

Market structure researchers have long-standing interest to examine the choice of informed traders, yet prior studies are usually abstract from the origin and the nature of their private information. In the shareholder activism literature, activists often have information that only they know. Therefore, we model their behavior as a strategic informed trader endogenizing the size of her order. This information structure differs from Hendershott and Mendelson (2000) and Zhu
(2014), who model the venue choice of infinitesimal informed traders. Not surprisingly, our model generates opposite predictions from theirs. For example, Zhu (2014) predicts that an informed trader is less likely to use a dark pool, whereas we predict that a large strategic informed trader trades more aggressively in the dark pool. Zhu (2014) predicts that the dark pool market share decreases with the value of private information as long as the informed trader uses the dark pool, whereas we predict that dark pool market share increases with the value of private information. The difference in predictions results from the nature of private information. Zhu (2014) features infinitesimal informed traders who have no impact on the price. As a result, the cost of non-execution becomes higher when the value of information increases. Therefore, informed traders tend to use the dark pool less, and an increase in the value of information reduces the incentive for informed traders to use the dark pool. However, our empirical results show that the dark pool volume increases more when traders have unique information and when they want to acquire a large position.

Our paper also contributes to the literature on rational expectation equilibrium. The workhorse model in this literature is the linear normal framework, in which agents follow a linear strategy and all noises are normally distributed (Grossman and Stiglitz, 1980; Kyle, 1985). To the best of our knowledge, all extensions of Grossman and Stiglitz (1980) and Kyle (1985) assume guaranteed execution of orders. The tractability offered by the linear normal model disappears when execution is uncertain. We propose a new way to analytically solve the rational expectation equilibrium with both price and execution uncertainty: the quadratic power-law framework. In this framework, the noise follows the power-law distribution and traders follow a quadratic strategy.

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1 Breon-Drish (2010) relaxes the assumption of the normality.
2 Interestingly, power-law distribution, the key assumption to generate tractability, is well supported by empirical evidence [see the two survey papers by Gabaix (2008, 2009)].
The closed-form solution under the quadratic power-law framework generates new economic insights, one of which is the possibility of price manipulation. Because dark pools use the price set by exchanges to match buy and sell orders, a report by the Congressional Research Service (2014) states that “HFT firms may be placing orders in the lit markets for the purpose of manipulating securities prices in dark pools.” Researchers have not examined price manipulation because they either assume a zero price impact (Hendershot and Mendelson, 2000; Degryse et al., 2009; Zhu, 2014; Buti et al., 2017) in which price manipulation is impossible, or they assume a price impact in which price manipulation is almost inevitable (Klöck et al., 2011). In the latter case, researchers need to assume that price manipulation does not exist (Kratz and Schöneborn, 2015).

Our model rules out price manipulation by endogenizing the price impact of trades. For example, Degryse et al. (2013) conjecture that manipulation risk is greater when the primary market is less liquid and the overall volume in the dark pool is larger. Yet we show that the informed trader has no incentive to trade in the opposite direction of her signal even if the liquidity trading in the dark pool is infinitely larger than the liquidity trading on the lit exchange. An increase in liquidity trading in the dark pool, in equilibrium, decreases the price impact of trades in the lit exchange. A reduction in the price impact decreases the cost for the informed trader to trade in the correct direction, but at the same time increases the cost to create mispricing in the exchange. When the liquidity trading in the dark pool approaches infinity, the price impact on the exchange approaches zero. The loss from creating mispricing in the lit exchange always outweighs the profit in the dark pool. Therefore, the informed trader always trades in the direction of her signal.

In this sense, we contribute to the literature on trade-based price manipulation, which aims to push the price up (down) through a purchase (sale), and can make a profit if the follow-on trades

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unwinding the opposition do not move the price back to the original level. Jarrow (1992) shows that trade-based manipulation exists only when the price impact is asymmetric. Huberman and Stanzl (2004) extend this intuition by showing that only the linear price impact functions rule out price manipulation. The literature on trade-based manipulation, therefore, focuses on the frictions that generate the asymmetric price impact of trades. The price impact in our model is neither symmetric nor linear: the informed trader can move the price up by buying in the lit exchange, and her trades to sell in the dark pool do not move the price. The absence of price manipulation relies on an endogenous price impact.

1. Institutional Background

In this section, we introduce the key institutional features of shareholder activism and dark pools that motivate our model setup and empirical research design.

1.1. Shareholder Activism through Schedule 13D Filings

Shareholder activism through governance intervention has emerged as a major force of corporate governance (Brav et al., 2008; Denes et al., 2017). The stated objectives in Schedule 13D filings of activist shareholders fall into five major categories: general undervaluation/maximize shareholder value, capital structure, business strategy, sale of target company, and corporate governance (Brav et al. 2008). The literature shows that activist intervention generates value for shareholders. For example, Collin-Dufresne and Fos (2015) show that market-adjusted return for the target firm is about 3% in the three-day window around the Schedule 13D filing date. Brav et al. (2015a) show that activists improve a firm’s productivity and Gantchev et al. (2017) show that

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these activists discipline not only managers of the targeted firms but also those of their competitors.

Our model is designed to capture two salient features of activist shareholders. First, activists are large traders, because they need to acquire a large stake in the target firm. Collin-Dufresne and Fos (2015) show that the mean (median) ownership that a Schedule 13D filer acquires by the initial filing date is 7.51% (6.11%). Second, trades executed by activists before the Schedule 13D filing date are based on their valuable private information. Such private information pertains to their own holdings in the target firms and planned governance agenda (Collin-Dufresne and Fos 2016). Activists do not need to report a trade until exceeding 5% ownership of the target firm (the event date), after which they have an additional 10 days to file a Schedule 13D with the SEC (the filing date). Consequently, pre-announcement trades by Schedule 13D filers can be classified as informed trades.

Collin-Dufresne and Fos (2015) show that Schedule 13D filers time their trades and use limit orders instead of marketable orders to minimize transaction costs. Yet no one has studied how activists minimize their transaction costs by choosing the trading venue. Specifically, activists can trade equities in more than 10 stock exchanges and 40 active dark pools. Anecdotal evidence suggests that institutional investors use dark pools to execute shares when they possess unique private information.\(^5\) However, these cases often involve litigations, the only scenario in which investors need to disclose where they execute trades. Our study provides systematic evidence on the venue choice of activist institutional investors.

1.2. Dark Pools

Dark pools have emerged as an important alternative trading venue. According to the SEC (2010) and TABB (2015), the market share of dark pools for U.S. listed stocks increased from about 8% in 2008 to 14% in 2015. SEC (2010), Zhu (2014), Buti et al. (2017), and Hasbrouck (2017) provide a detailed summary of the pricing mechanism, matching frequency, and allocation rule of the three major types of dark pools: crossing networks (e.g., ITG POSIT), nondisplayed limit-order-book (e.g., GS Sigma X), and PING destinations (e.g., Knight Link).

Despite heterogeneity in trading mechanisms, dark pools share two important commonalities. First, dark pools often rely on benchmark prices derived from lit exchanges to match buy and sell orders. At the minimum, the dark pools use prices derived from lit exchanges as a benchmark. The benchmark price can be the midpoint between the national best bid and offer prices (NBBO), the volume-weighted average price (VWAP), and the closing price (Hasbrouck, 2007). Trades in dark pools, however, do not have a direct impact on the benchmark prices. Some dark pools choose benchmark prices after the quantity match. For example, ITG’s POSIT conducts scheduled matches at random times within periodic windows. A random match time discourages traders from manipulating the benchmark price to obtain a favorable price. Some dark pools choose a benchmark price before the quantity match. For example, the INSTINET’s closing cross allows traders to submit orders after the regular market closes. These orders will be matched and executed at the closing price. Because price is determined before the quantity match, dark pools take actions to discourage predatory trading. For example, INSTINET cancels crosses when there are news announcements and expel participants’ strategies that appear to be news-driven.

Second, orders in dark pools do not have pre-trade transparency, that is, other market participants usually cannot see the size of the orders. Conditional on execution, the transaction costs in dark pools are lower as trades do not directly move prices. However, dark pools have
lower execution probability. One reason is that the prices in dark pools are not market-clearing prices. Instead, they are derived from the lit exchange. Therefore, prices play a minor role in balancing the buy and sell orders in dark pools. Ye (2010) finds that the average execution probability for dark pools is only 4.11% for NYSE stocks and 2.17% for NASDAQ stocks, whereas Tuttle (2013) finds that only 0.69% of shares routed to dark pools are filled. The above comparison suggests that investors face a trade-off between the price impact and execution probability when choosing to trade in dark pools instead of lit exchanges.

While the original purpose of creating a dark pool is to facilitate large liquidity trades by institutional traders, the benefit of anonymity also attracts informed traders. There are great concerns among regulators and practitioners that informed traders can create toxic order flows or even manipulate prices to profit from dark pools. Our results show that the endogenous price impact of trade would mitigate incentives for price manipulation.

2. Model

In this section, we develop a two-period model of a trading venue choice for a large informed trader like an activist institutional shareholder. In Section 2.1, we set up the model. In Section 2.2, we define the rational expectation equilibrium of the model.

2.1. Model Setup

Our model is a variation of Kyle’s (1985) canonical strategic trading model. We consider a two-period framework with two markets: a lit exchange and a dark pool. A single risky asset is traded by three types of agents: a risk-neutral informed trader, many liquidity traders, and a market maker.

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6 In 2009, the SEC accused the Galleon Group of hedge funds of trading on inside information, when “tens of millions of shares move each day through ‘dark pools,’ where quotes aren't displayed until after the trade is done.” See “Staying Calm in a World of Dark Pools, Dark Doings,” The Wall Street Journal, October 4, 2009.
The asset has a stochastic liquidation value $\tilde{v}_g$ with $E(\tilde{v}_g) = p_0$. The informed trader observes in advance the realization of $\tilde{v}_g$, denoted $v_g$, and submits $x_e \in \mathbb{R}$ to the lit exchange and $x_d \in \mathbb{R}$ to the dark pool to maximize the value of the information.

As in Collin-Dufresne and Fos (2016), liquidity traders are passive players in our model. In Collin-Dufresne and Fos (2016), liquidity traders do not choose their trades across time; in our model, they do not choose between venues. We build the first tractable rational expectation equilibrium model that has both price uncertainty and execution uncertainty, but the tractability disappears when liquidity traders are able to choose venues. This limitation makes us focus our predictions on the environment that is closest to our assumptions: shareholder activism. Also, recent empirical evidence suggests that liquidity traders often delegate the venue choice to brokers, and brokers often violate responsibility of best execution by routing orders to their own dark pools or exchanges that offer the highest rebates (Battalio et al., 2016; Anand et al., 2019). Finally, our simplification of exogenous liquidity traders also strengthens our economic insight on price manipulation. We show that, once we endogenize the price impact, price manipulation is impossible even when liquidity traders cannot choose their markets and their trading volume in the dark pool is infinitely large relative to their volume in the lit exchange.

The liquidity order flow in the lit exchange is denoted as $\tilde{u}_e \in \mathbb{R}$. Let $\tilde{u}^b_d \in \mathbb{R}^+$ and $\tilde{u}^s_d \in \mathbb{R}^+$ be the unsigned aggregate liquidity buy and sell order flows in the dark pool, respectively. $\tilde{v}_g$, $\tilde{u}_e$, $\tilde{u}^b_d$, and $\tilde{u}^s_d$ are independently distributed.

Figure 1 shows the timeline of our two-period model. At time 0, all four random variables are realized. The informed trader observes $v_g$ but not $\tilde{u}_e$, $\tilde{u}^b_d$, or $\tilde{u}^s_d$. Her trading strategy $\{X_e, X_d\}$ assigns order sizes in the lit exchange and the dark pool for each $v_g$. The dark pool opens in the second period, but it only accepts orders before time 1. Therefore, the informed trader cannot
compare the realizations of $\tilde{p}$ and $\tilde{v}_g$ and conduct predatory trading. At time 1, when the lit exchange opens, the market maker observes the aggregate order flow in the exchange, $\tilde{y} = \tilde{x}_e + \tilde{u}_e$, but not $\tilde{v}_g$, $\tilde{u}_e$, $\tilde{u}^b_d$, or $\tilde{u}^s_d$. The market maker's pricing rule is $P$, which assigns to each outcome of $\tilde{y}$ a semi-strong efficient price $\tilde{p}$ based on his conjecture of the informed trader's strategy $\{X_e, X_d\}$. At time 2, the dark pool uses $\tilde{p}$ to match buy and sell orders. The side with smaller aggregated order size gets full execution, while the side with larger aggregated order size gets partial execution. The stock liquidates at the end of period 2.

*Insert Figure 1 Here*

As in Kyle’s (1985) model, in our model the liquidity order flow in the lit exchange follows normal distribution, $\tilde{u}_e \sim N(0, \sigma_e^2)$. The standard deviation $\sigma_e$ serves as a proxy for the level of liquidity trading in the exchange.\(^8\)

In the dark pool, the unsigned liquidity buy, $\tilde{u}^b_d$, and the unsigned liquidity sell, $\tilde{u}^s_d$, follow power-law distributions. Just as the normality generates the tractability of the rational expectation equilibrium model when execution is certain, power-law is essential to generate the tractability when price and execution are both uncertain. The model intuition does not rely on the power-law distribution, although tractability depends on this assumption. Tractability is essential for models in the rational expectation equilibrium framework. Without tractability, it is hard to prove whether the equilibrium exists, let alone characterizing it (O’Hara, 1995).

The cumulative distribution function (CDF) of $\tilde{u}^s_d$ is:

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\(^7\) In reality, a dark pool often has technology to prevent traders from knowing the exact matching time and price (Hasbrouck, 2007).

\(^8\) The unsigned order flow, $|\tilde{u}_e|$, follows a folded normal distribution, with $E|\tilde{u}_e| = \sqrt{\frac{2}{\pi}} \sigma_e$. Thus, the expected size of the uninformed order flow is linear in $\sigma_e$. 

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\[ F_s(z; k) = P(\tilde{u}_d^s \leq z) = \begin{cases} 
0, & \text{for } z < 0 \\
1 - \frac{k}{\sqrt{z + k}}, & \text{for } z \geq 0
\end{cases}, \quad (1) \]

which also implies the following probability distribution function (PDF) of \( \tilde{u}_d^s \):

\[ f_s(z; k) = \begin{cases} 
0, & \text{for } z < 0 \\
\frac{1}{2} k \frac{1}{(z + k)^{3/2}}, & \text{for } z \geq 0
\end{cases}. \quad (2) \]

\( \tilde{u}_d^b \) follows similar definition.

Parameter \( k \), the scale, captures the level of liquidity trading in the dark pool. Figure 2 shows that a distribution with a higher \( k \) first order stochastically dominates a distribution with a lower \( k \). Therefore, the level of liquidity trading in the dark pool increases with \( k \). As one focus of our paper is price manipulation, we consider the case where the informed trader has the highest incentive to create mispricing. We assume that the orders from the informed trader have higher execution priority over liquidity traders. Our model becomes highly intractable under the pro rata rule, because such a rule implies that we need to calculate the expectation of the ratio of two random variables. Fortunately, under the power-law distribution, our qualitative results should still hold with alternative allocation rules, because these alternative allocations rules simply reduce \( k \), or the liquidity trading available to the informed trader. All the economic mechanisms in the model remain the same.

\textit{Insert Figure 2 Here}

Following Seppi (1997), Hendershott and Mendelson (2000), Parlour and Seppi (2003), and Foucault and Menkveld (2008), we assume an up-front order submission cost of \( c \) per share. The submission cost is a standard assumption to prevent orders of infinite size. The model holds for any positive \( c \), so its value can be arbitrarily small; \( c \) applies to both the lit exchange and the
dark pool.⁹

The informed trader's profit per unit is \((v_g - \bar{p}) - c\) when she buys and \((\bar{p} - v_g) - c\) when she sells. The cost \(c\) increases the fundamental value to the informed buyer and decreases the fundamental value to the informed seller. When \(v_g \in [p_0 - c, p_0 + c]\), the potential revenue of trading is lower than the order submission cost, and the informed trader neither buys nor sells. It is hard to incorporate this non-trading area into the equilibrium even if there is only a lit exchange [see Bernhardt and Hughson (2002) for the solution]. To rule out the possibility of non-trading, Hendershott and Mendelson (2000) assume that informed traders always trade. We replace their assumptions with a rigorous proof. For this purpose, we define a new variable, \(\tilde{v} \sim N(p_0, \sigma_v^2)\). Then, we define \(\tilde{v}_g\) as the following transformation of \(\tilde{v}\):

\[
\tilde{v}_g = \begin{cases} 
\tilde{v} + c & \text{when } \tilde{v} \geq p_0 \\
\tilde{v} - c & \text{when } \tilde{v} < p_0.
\end{cases}
\] (3)

The only purpose for equation (3) is to make our proof mathematically rigorous. The transformation in equation (3) means that the value of the information is normally distributed after deducting the order submission cost. The major steps of the proof focus on \(\tilde{v}\), because the discontinuity of \(\tilde{v}_g\) cancels with \(c\). In addition, all the results hold even if \(c\) approaches 0. Equation (3) implies that \(\Pr(\tilde{v}_g \in [p_0 - c, p_0 + c]) = 0\); thus, the information always leads the informed trader to submit orders. Finally, \(\tilde{v}_g\) and \(\tilde{v}\) are informationally equivalent because they have a one-to-one mapping, such that \(E(.|v_g) = E(.|v)\). We also assume that the order submission cost

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⁹ Because all orders in the exchange are executed, \(c\) can be interpreted as the commission or the fee paid by the informed and liquidity traders to access the market. One interpretation to explain the order submission cost in the absence of guaranteed execution is that it "capture any incremental opportunity or shoe leather costs investors bear when trading from off the exchange" (Seppi, 1997). Another way to understand variable cost \(c\) is related to anti-gaming strategy in the dark pool. The dark pool gives lower trading priority for orders with larger size because the order is more likely from an informed trader. This is certainly a cost to submitting a large order, which is captured by \(c\).
compensates the market maker so that he makes decisions conditional on $E(\cdot|v)$.

2.2. Definition of the Equilibrium

The definition of equilibrium is as follows:

**Definition 1**: A rational expectation equilibrium is an informed order submission strategy $X_e, X_d$, and a market maker’s pricing rule $P$, such that the following two conditions hold:

1. **Profit Maximization**: For any alternative strategy $X_e', X_d'$, and any realization $v$ (or $v_g$):
   \[
   E[\tilde{\pi}(X_e, X_d, P|\tilde{v} = v)] \geq E[\tilde{\pi}(X_e', X_d', P|\tilde{v} = v)];
   \]

2. **Market Efficiency**: The random variable $\tilde{p}$ satisfies:
   \[
   \tilde{p}(X_e, X_d, P) = E[\tilde{v}|\tilde{y} = \tilde{x}_e + \tilde{u}_e].
   \]

Again, the market efficiency condition implies $\tilde{p} = E[\tilde{v}|\tilde{y}]$ instead of $\tilde{p} = E[\tilde{v}_g|\tilde{y}]$ because we assume that the order submission cost compensates the market maker.

3. Equilibrium

In this section, we solve for the equilibrium $X_e$ and $X_d$, as well as the pricing rule $P$, by guessing and verifying. We guess that $x_e = X_e(v) = \alpha + \beta v$ and $P(y) = \mu + \lambda y$. We will show later that $X_e(v)$ and $P(y)$ define the unique $X_d(v)$.

3.1. Expected Profit

The informed trader maximizes the expected profit. Since informed buying and selling are separable and symmetric in the model, we focus on the case in which $v_g \geq p_0 + c (v \geq p_0)$.

Suppose that the informed trader chooses $\{x_e, x_d\}$ to buy. She then expects that the market maker will set the price at $\tilde{p} = \mu + \lambda (x_e + \tilde{u}_e)$. Therefore, her expected profit in the exchange is:

\[
E(\tilde{\pi}_e) = E \left[ (v_g - \mu - \lambda (x_e + \tilde{u}_e))x_e - cx_e \right] = (v - \mu - \lambda x_e)x_e. \tag{4}
\]
The informed trader’s expected profit in the dark pool is much more complex. Executed shares $\bar{x}_m$ depend on $x_d$, as well as on the realizations of the liquidity sell order. $x_d$ gets full execution if it is smaller than $\bar{u}_d^s$; otherwise, only $\bar{u}_d^s$ shares get executed. Therefore,

$$\bar{x}_m(\bar{u}_d^s, x_d) = \begin{cases} \bar{u}_d^s & \text{when } \bar{u}_d^s \leq x_d \\ x_d & \text{when } \bar{u}_d^s > x_d. \end{cases} \quad (5)$$

The expected number of executed shares conditional on $x_d$ is:

$$E(\bar{x}_m | x_d) = \int_0^{+\infty} \bar{x}_m(\bar{u}_d^s, x_d) f_s(z) dz = \int_0^{x_d} z f_s(z) dz + x_d \int_{x_d}^{+\infty} f_s(z) dz. \quad (6)$$

Lemma 1 states the relation between the number of submitted shares and the executed shares.

**Lemma 1**: $E(\bar{x}_m | x_d)$ is increasing in $x_d$, and if $x_d \neq 0$, the probability of execution $\frac{E(\bar{x}_m | x_d)}{x_d}$ decreases with $x_d$.

**Proof**: See Appendix A.

Lemma 1 captures the probability impact of the trade. As the informed trader increases the order size in the dark pool, the expected volume increases but the execution probability decreases. Therefore, an increase in order size leads to an increase in total expected revenue but a decrease in the expected marginal revenue. Although the power-law framework is needed to analytically solve the model, the intuition of the probability impact of the trade holds as long as $f_s(z) > 0$, and almost surely when $z \in [0, +\infty)$.

As the dark pool matches orders based on the price in the lit exchange, an informed trader’s revenue per matched share is $v_g - \mu - \lambda(x_e + \bar{u}_e)$. Her total revenue is $\left( v_g - \mu - \lambda(x_e + \bar{u}_e) \right) \bar{x}_m(\bar{u}_d^s, x_d)$, and her expected profit in the dark pool is:

$$E(\bar{\pi}_d) = E\left[ \left( v_g - \mu - \lambda(x_e + \bar{u}_e) \right) \bar{x}_m(\bar{u}_d^s, x_d) - cx_d \right]$$
\[ = (v_g - \mu - \lambda x_e) E(x_m | x_d) - cx_d \]
\[ = (v_g - \mu - \lambda x_e) \left( \int_0^{x_d} zf_s(z) dz + x_d \int_{x_d}^{+\infty} f_s(z) dz \right) - cx_d. \quad (7) \]

The informed trader chooses \( \{x_d, x_e\} \) to maximize her two-period profit. That is,

\[ \max_{x_d, x_e} E(\bar{\pi}) = E(\bar{\pi}_e + \bar{\pi}_d) \]
\[ = (v - \mu - \lambda x_e) x_e + (v_g - \mu - \lambda x_e) \left( \int_0^{x_d} zf_s(z) dz + x_d \int_{x_d}^{+\infty} f_s(z) dz \right) - cx_d. \quad (8) \]

### 3.2. Equilibrium Order Submission Strategy and Pricing Rule

Equation (8) contains two first-order conditions. The first-order condition with respect to \( x_d \) is:

\[ \frac{\partial E(\bar{\pi})}{\partial x_d} = (v_g - \mu - \lambda x_e) \int_{x_d}^{+\infty} zf_s(z) dz - c = 0 \]
\[ \Rightarrow (v_g - \mu - \lambda x_e)(1 - F_s(x_d)) = c \]
\[ \Leftrightarrow (v_g - \mu - \lambda x_e) Pr(\tilde{u}_d > x_d) = c. \quad (9) \]

The term \((v_g - \mu - \lambda x_e)\) represents the informed trader’s revenue for each matched share, and \(Pr(\tilde{u}_d > x_d)\) is the probability that the \(x_d^{th}\) unit is executed. The informed trader chooses \(x_d\), such that the expected marginal revenue \((v_g - \mu - \lambda x_e) Pr(\tilde{u}_d > x_d)\) is equal to the order submission cost cost \(c\).

The first-order condition with respect to \(x_e\) is:

\[ \frac{\partial E(\bar{\pi})}{\partial x_e} = v - \mu - \lambda x_e - \lambda \left( \int_0^{x_d} zf_s(z) dz + x_d \int_{x_d}^{+\infty} f_s(z) dz \right) = 0 \]
\[ \Leftrightarrow v - \mu - \lambda x_e = \lambda x_e + \lambda E(x_m | x_d). \quad (10) \]

Equation (10) captures the externality of the price impact in the lit exchange on the dark pool. The margin gain to buy one more share in the exchange is \(v - \mu - \lambda x_e\). Buying one more share, however, leads the market maker to increase the price by \(\lambda\). In Kyle’s (1985) model, the
informed trader loses $\lambda x_e$ and the first-order condition is $v - \mu - \lambda x_e = \lambda x_e$. In our model, the dark pool also uses the price set by the lit exchange to match orders. Therefore, equation (10) contains a new term, $\lambda E(\tilde{x}_m|x_d)$, because the informed trader’s order flow in the lit exchange creates externality for her order flow in the dark pool. This externality is the key mechanism that drives our theoretical mechanisms and empirical predictions.

The power-law distribution helps us to obtain the analytical solution for equations (9) and (10). First, we plug the CDF of the power-law distribution into equation (9) as follows:

$$(v_g - \mu - \lambda x_e)(1 - (1 - \frac{k}{\sqrt{x_d + k}})) = c$$

$\iff (v_g - \mu - \lambda x_e)\frac{k}{\sqrt{x_d + k}} = c.$

Therefore,

$$x_d^* = \frac{k}{c^2}(v + c - \mu - \lambda x_e)^2 - k. \quad (11)$$

When $v_g \leq p_0 - c$ or $v \leq p_0$, $x_d^* = -\frac{k}{c^2}(\mu + \lambda x_e - v_g)^2 + k = -\frac{k}{c^2}(\mu + \lambda x_e - v + c)^2 + k$. Lemma 2 shows that the discontinuity created for $v_g$ cancels out with $c$. The expected executed shares, $E(\tilde{x}_m|x_d^*)$, the expected profit in the dark pool, $E(\tilde{\pi}_d^*)$, and the informed trader’s strategy, $x_d^*$, are continuous functions of $v$.

**Lemma 2.** The informed trader's order submitted to the dark pool is:

$$\begin{cases} 
  x_d^* = \frac{k}{c^2}(v + c - \mu - \lambda x_e)^2 - k & \text{for } v \geq p_0 \\
  x_d^* = -\frac{k}{c^2}(\mu + \lambda x_e - v + c)^2 + k & \text{for } v < p_0 
\end{cases}$$

which implies: $E(\tilde{x}_m|x_d^*) = 2\frac{k}{c}(v - \mu - \lambda x_e)$ and $E(\tilde{\pi}_d^*) = \frac{k}{c}(v - \mu - \lambda x_e)^2$.

**Proof:** See Appendix A.
Lemma 2 shows that the informed trader’s optimal strategy $x_d^*$ is a quadratic function of $v$ and $x_e$. The executed shares, however, increase linearly with $v$ and $x_e$ due to the non-execution probability. The expected profit in the dark pool then becomes a quadratic function of $v$ and $x_e$. As the informed trader’s profit in the exchange is also a quadratic function of $v$ and $x_e$, her total profit in both trading platforms is quadratic in $v$ and $x_e$. The quadratic profit function then makes the optimal $x_e$ linear in $v$. Therefore, the quadratic power-law framework allows us to extend the rational expectation equilibrium literature to markets with execution uncertainty.

To be more specific, let us substitute $E(x_m|x_d)$ into the first-order condition equation (10), giving:

$$v - \mu - \lambda x_e - \lambda x_e - 2\lambda \frac{k}{c} (v - \mu - \lambda x_e) = 0. \quad (12)$$

Define $K \equiv (k/c)$. The optimal value of $x_e$ is:

$$x_e^* = \frac{1 - 2\lambda K}{2\lambda - 2\lambda^2 K} v - \frac{1 - 2\lambda K}{2\lambda - 2\lambda^2 K} \mu. \quad (13)$$

Comparing the coefficient with the conjecture $x_e = \alpha + \beta v$ yields:

$$\beta = \frac{1 - 2\lambda K}{2\lambda - 2\lambda^2 K} \quad (14)$$

$$\alpha = -\beta \mu. \quad (15)$$

The market maker then sets the clearing price equal to the conditional expectation of $v$:

$$p = \mu + \lambda y = E(\tilde{v}|x_e + \tilde{u}_e = y) = E(\tilde{v}|\alpha + \beta \tilde{v} + \tilde{u}_e = y). \quad (16)$$

Because $\tilde{v}$ and $\tilde{u}_e$ are normally distributed, the Projection Theorem yields:

$$\lambda = \frac{Cov(\tilde{v}, \tilde{y})}{Var(\tilde{y})} = \frac{\beta \sigma_\tilde{v}^2}{\beta^2 \sigma_\tilde{v}^2 + \sigma_\tilde{u}^2} = \frac{\beta}{\beta^2 + \sigma_\tilde{u}^2 / \sigma_\tilde{v}^2} = \frac{\beta}{\beta^2 + R} \quad (17)$$

$$\mu = p_0 - \lambda (\alpha + \beta p_0), \quad (18)$$

where $R \equiv \frac{\sigma_\tilde{u}^2}{\sigma_\tilde{v}^2}$. Combining (15) and (18) yields:
\[(\lambda \beta - 1)(p_0 - \mu) = 0. \quad (19)\]

From equation (17), \(\lambda \beta = (\beta^2)/(\beta^2 + R) < 1\), which implies that:

\[\mu = p_0 \text{ and } \alpha = -\beta p_0. \quad (20)\]

\(\lambda\) can then be solved by substituting (14) into (17) to give:

\[
\lambda = \frac{1 - 2\lambda K}{2\lambda - 2\lambda^2 K} \Rightarrow \frac{1 - 2\lambda K}{(2\lambda - 2\lambda^2 K)^2 + R} \Rightarrow R(2\lambda - 2\lambda^2 K)^2 = (1 - 2\lambda K). \quad (21)
\]

\(\beta\) is uniquely defined by (14) for any \(\lambda\). In turn, \(\alpha\) is uniquely defined by (15). Then the key to solving the model is solving (21). The Ferrari method guarantees closed form solutions for equation (21) because it is a depressed quartic equation in \(\lambda\).\(^{10}\) We do not present these solutions because they are overwhelmingly complex in \(R\) and \(K\). It is more convenient to prove the existence and uniqueness of the solution and then conduct comparative statics by analyzing (14) and (21).

As a benchmark, our model degenerates into the Kyle (1985) model if the dark pool does not exist. The solutions for \(K = 0\) are \(\beta = \sqrt{R} = \frac{\sigma_e}{\sigma_v}\) and \(\lambda = (1/(2\sqrt{R})) = \frac{\sigma_e}{2\sigma_v}\), which are the same as in a one-period Kyle model.

3.3. Unique Linear Equilibrium and Price Manipulation

Theorem 1 establishes the existence and uniqueness of the solution for our model.

**Theorem 1.** There exists a unique linear equilibrium in which: the informed trader submits

\[X_e(v) = \beta^*(v - p_0), \text{ and } X_d(v) = \begin{cases} 
\frac{k}{c^2} (v + c - p_0 - \lambda^* x_e)^2 - k & \text{for } v \geq p_0 \\
-\frac{k}{c^2} (p_0 + \lambda^* x_e - v + c)^2 + k & \text{for } v < p_0
\end{cases}, \text{ and the price function is } P(y) = p_0 + \lambda^* y. \ \lambda^* \text{ is the unique positive solution of } R(2\lambda^* - 2\lambda^{*2}K)^2 = (1 - \]

\(^{10}\) A depressed quartic equation is a quartic equation with no cubic term. In the sixteenth century, Ferrari found the formula to express the solution of any depressed quartic equation in terms of its coefficients.
2\(\lambda^*K\)) and \(\beta^* = \frac{1-2\lambda^*K}{2\lambda^*-2\lambda^{*2}K} > 0\), where \(R \equiv \frac{\sigma^2_e}{\sigma^2_v} > 0\) and \(K \equiv \frac{k}{c} \geq 0\).

**Proof:** See Appendix A.

As \(\beta^* > 0\), the informed trader always trades in the correct direction of her signal in the unique linear equilibrium. For any value of \(k\) or \(\sigma^2_e\), the informed trader has no incentive to move the price in the wrong direction and benefits from the mispricing by matching orders in the dark pool. This result is surprising, especially when \(k\) is large or when \(\sigma^2_e\) is small. A deep dark pool seems to create a large benefit for price manipulation, and a thin exchange seems easy to manipulate. However, Theorem 1 implies that the cost of price manipulation always outweighs the benefit.

Corollary 1 shows that an endogenous price impact explains this surprising result. An increase in liquidity trading in the dark pool increases the benefit of price manipulation, but it also increases its costs, because the price impact decreases with the liquidity volume in the dark pool. Therefore, the informed trader finds it harder to move the price in the wrong direction with a deeper dark pool, although the benefit from the same level of mispricing is higher.

**Corollary 1: Price impact.** \(\lambda^*\) decreases in \(k\) and \(\sigma_v\) and increases in \(\sigma_v\) and \(c\). \(\lambda^* \in \left(\frac{0}{2K}, 1\right)\).

As \(K \to +\infty\), \(\lambda^* \to 0\).

**Proof:** See Appendix A.

Corollary 1 shows that when liquidity trading in the exchange approaches infinity, the price impact in the exchange approaches 0, implying that it is extremely difficult to create mispricing. In models with exogenous price impact, such as models that follow the Almgren-Chriss framework ((Kratz and Schöneborn, 2015; Klöck et al., 2011), price manipulation is inevitable. We rule out the possibility of price manipulation by endogenizing the price impact.

Corollary 2 shows the comparative statics of \(\beta^*\).
**Corollary 2: Informed trader’s choice:** The informed trader trades in both markets unless \( v = p_0 \). \( \beta^* \) increases in \( \sigma_e \) and \( c \) and decreases in \( k \) and \( \sigma_v \); the magnitude of the informed trader's order in the dark pool, \( |x_d^*| \), increases in \( k \) and \( \sigma_v \) and decreases in \( \sigma_e \) and \( c \).

**Proof:** See Appendix A.

Corollary 2 first shows that the informed trader always trades in both markets, and this result can be proved intuitively through contradiction. Suppose that the informed trader did not trade in the lit exchange, then the price impact would be zero. Then the informed trader would trade in the lit exchange if the price impact was zero, which is a contradiction. Similarly, the non-execution probability would be zero if the informed trader did not trade in the dark pool. Therefore, the informed trader always trades in the dark pool when \( v \neq p_0 \).

Corollary 2 suggests that the informed trader trades less aggressively in the lit exchange when there is a dark pool, because the price impact of the trade in the exchange imposes a negative externality on her profit in the dark pool (equation (10)). Therefore, \( \beta^* \) decreases, and the order flow becomes less informative. The informed trader trades more aggressively in the exchange when the liquidity trading in the exchange is high. When the liquidity trading in the dark pool is high, the externality of the price impact is high, and the informed trader trades less aggressively in the exchange. An increase in the fundamental value uncertainty, \( \sigma_v \), increases the informed trader's order size in the dark pool but decreases her order aggressiveness in the lit exchange, because she has a greater incentive to hide in the dark pool when the value of information is high.

4. **Empirical Predictions**

In this section, we summarize the empirical predictions of our model.

4.1. **Order Splitting**
The key prediction of our model relates to the order-splitting behavior of the informed trader. In equilibrium, the informed trader submits $x_e^*$ to the exchange and $x_d^*$ to the dark pool. The market share of the dark pool, however, depends on $E(\tilde{x}_m|x_d^*)$ because only part of the order is executed.

In Theorem 2, we summarize the order-splitting strategy.

**Theorem 2.** $\frac{E(\tilde{x}_m|x_d^*)}{x_e^*} = 1 - \frac{1}{1-2\lambda^*K'} \cdot \frac{E(\tilde{x}_m|x_d^*)}{x_e^*}$ is uniquely determined by $\left(\frac{k\sigma_v}{\sigma_e^c}\right)^2$ and increases with $\left(\frac{k\sigma_v}{\sigma_e^c}\right)^2$.

**Proof:** See Appendix A.

As liquidity traders are passive players in our model, Theorem 2 has implications for both the order splitting behavior of the informed trader and the market share of the dark pool. Our empirical predictions focus on the latter as the former is not empirically observable. Also, it is well accepted that an abnormal pattern in total trading volume during times of informed trading reflects the order submission strategy of the informed trader (e.g., Chae, 2005; Graham et al., 2006; Sarkar and Schwartz, 2009; Baruch et al., 2017). Theorem 2 shows that the market share of the dark pool should increase with the value of private information. To be more specific, Theorem 2 implies that the market share of the dark pool is lowest when there is no informed trader ($\sigma_v = 0$). The presence of an informed trader increases the market share of the dark pool, because an informed trader tends to trade more in the dark pool than in the lit exchange. Theorem 2 also implies that, conditional on the presence of an informed trader, the market share of the dark pool increases more when the value of information is higher.

Our predictions are in the opposite direction to models in which trades do not move price. Zhu (2014) predicts that infinitesimal informed traders are less likely to use dark pools. In some parameters, informed traders may completely avoid dark pools. When informed traders use dark pools, Zhu (2014) predicts that the market share of the dark pool decreases with the value of private
information, because the cost of non-execution is higher when the value of information is high. Menkveld et al. (2017) model the venue choice of uninformed liquidity traders who trade off the bid-ask spread on lit exchanges and the delay cost of non-execution in dark pools, both of which are exogenously given. As their trades do not move the price, liquidity traders prefer the lit exchange when the value of information increases, as the delay cost of non-execution in the dark pool is higher when the value of information is higher.

Theorem 2 also shows that an increase in $\sigma_e$ leads the informed trader to submit relatively more shares to the lit exchange, whereas an increase in $k$ makes her submit relatively more shares to the dark pool. If $\sigma_e$ and $k$ increase at the same rate, the informed trader maintains the ratio of trades made in these two markets. Interestingly, Theorem 2 shows that the value of information $\sigma_v$ and $\frac{k}{\sigma_e}$ (the liquidity trading in the dark pool relative to liquidity trading on the lit exchange) have the same impact on the dark pool’s market share, because they affect the market share of the dark pool through their product. The exact quantitative relationship certainly depends on the functional form in our model, but the intuition should hold more generally. More specifically, the result comes from the externality of the lit exchange’s price impact on the dark pool. The informed trader has a higher incentive to hide her information when the value of information is high, or when dark pool contains more liquidity trading. The above discussions of Theorem 2 lead to two empirical predictions.

**Prediction 1 (P1):** Dark pool market share (i.e., the percentage of total trading volume executed in dark pools) increases when a large informed trader trades.

**Prediction 2 (P2):** The increase in dark pool market share on days with informed trades is larger when the value of the private information possessed by the informed trader is larger and when relative liquidity in the dark pool is higher (i.e., liquidity trading in dark pools compared with that
4.2. Price Discovery

The order-splitting behavior of the informed trader leads to predictions on price discovery. We caution that our predictions should apply to an environment where one large trader possesses unique private information. With a dark pool ($K > 0$), our model generates two predictions that are different from Kyle’s (1985). In the one-period Kyle model, price reveals one-half of the informed trader’s information, and the information revelation is independent of the level of liquidity trading and fundamental value uncertainty. Figure 3 shows that: (1) with a dark pool, the price reveals less than half of the information; and (2) the level of information revelation increases with the level of liquidity trading in the lit exchange and decreases with the liquidity trading in the dark pool and fundamental value uncertainty. When $K \equiv k/c = 0$, $e$ reaches its maximum value of 0.5, and the result holds for any value of $R$, as in the Kyle’s (1985) model. When $K > 0$, $e < 0.5$ for any value of $R$, so that the dark pool makes the price less informative.

Insert Figure 3 Here

In Theorem 3, we summarize the impact of four exogenous variables ($k, \sigma_v, \sigma_e, and c$) on price discovery.

**Theorem 3.** The price informativeness measure, $e$, is uniquely determined by $\frac{k\sigma_v}{\sigma_e c}$ and decreases in $\left(\frac{k\sigma_v}{\sigma_e c}\right)^2$. When $k=0$, $e$ reaches its maximal value of $\frac{1}{2}$ and is independent of $\sigma_v$, $\sigma_e$, and $c$.

**Proof:** See Appendix A.

In our model, a dark pool harms price discovery, which contrasts with Zhu’s (2014) prediction. One reason for the difference comes from the strategy of the informed trader. Zhu
(2014) has infinitely many small informed traders who want to trade one unit of security. Each informed trader has no price impact of trade. In our model, the large informed trader is able to endogenize the size of a trade and she can split trades between the lit exchange and the dark pool. As the informed trader’s order in the lit exchange moves the price, the externality of the price impact of the trade from the exchange to the dark pool makes her trade less in the exchange. The limitation of our model is that we do not consider the venue choice of the liquidity traders. Therefore, the best we can claim is that our model captures one economic channel in which a dark pool harms price discovery, whereas Zhu (2014) captures a channel in which a dark pool improves price discovery. These competing effects may explain why the findings on price discovery and dark pool are mixed (Comerton-Forde and Putniņš, 2015; Foley and Putniņš, 2016; Hatheway et al., 2017). To the best of our knowledge, no existing models can endogenize both the size and venue choices of the traders.

For tractability, we assume that liquidity trading in both the lit exchange and the dark pool follows different distributions. Therefore, this model cannot hold fixed the total mass of liquidity trading after adding a dark pool. However, the second part of Theorem 3 suggests that the result should be robust no matter whether the dark pool attracts new liquidity trading or the dark pool attracts liquidity trading from the lit exchange. The liquidity trading in the lit exchange and the dark pool affects price discovery through their relative size $\frac{k}{\sigma_e}$. When there is no dark pool, $\frac{k}{\sigma_e} = 0$ and price discovery is maximized. If the dark pool attracts new liquidity trading, $k$ becomes positive when holding $\sigma_e$ fixed, which increases $\frac{k}{\sigma_e}$ and reduces price discovery. If the dark pool attracts liquidity traders from the lit exchange, then $\sigma_e$ must decrease when $k$ increases. Again, $\frac{k}{\sigma_e}$ increases and dark pool harms price discovery. It is worth noting that the price discovery in Kyle (1985) is independent of liquidity trading. In our model, liquidity trading in the dark pool reduces
price discovery as it allows the informed trader to execute orders without directly moving the price.

Liquidity trading levels in the dark pool and fundamental value uncertainty have equivalent effects on price discovery through their products. Figure 3 shows that the relation between \( k \) and price discovery is identical to the relation between \( \sigma_v \) and price discovery. An increase in \( k \) implies that the dark pool creates more “space” for the informed trader to hide information. An increase in \( \sigma_v \) increases the value of the information for the informed trader and leads to a greater incentive to hide. Both effects, however, result in less informative order flow in the lit exchange, thereby reducing the price discovery.

The above discussion of Theorem 3 leads to our third empirical prediction.

**Prediction 3 (P3):** Price discovery (i.e., one minus the fraction of private information incorporated into the stock price after announcement of informed trades) decreases with the value of private information and the relative liquidity trading in the dark pool.

5. Sample Description

In this section, we describe our sample and data. In Subsection 5.1, we describe our sample of Schedule 13D events. In Subsection 5.2, we outline the datasets we use to construct the market share of the dark pool, the measure of price discovery, and key explanatory variables. In Subsection 5.3, we construct the matched sample of control stocks. In Subsection 5.4, we present the summary statistics for our sample.

5.1. Schedule 13D Events

We start with 1,164 Schedule 13D filings from 2014 to 2018 in WRDS SEC Analytics Suite that target common stocks (i.e., CRSP share code 10 or 11) listed on the NYSE, AMEX, and NASDAQ. Among them, 642 filings disclose the date on which aggregate beneficial ownership exceeds the
reporting threshold of 5% (i.e., the event date).\textsuperscript{11} We exclude four filings that are filed more than 10 business days after the event date and 36 filings that are preceded by another filing targeting at same firm in the prior 120 trading days. Because our dark pool volume data become available after May 2014 and are at weekly frequency, we also exclude 66 Schedule 13Ds filed before May 2014 and 118 filings for which event date and filing date are in the same week. The latter filter separates trades based on private information (i.e., on event day) from trades triggered by public information (i.e., on filing day). For the final sample of 418 events, we collect filing date, event date, aggregate beneficial ownership at the filing date, open market transaction date, transaction type (buy or sell), transaction size, and transaction price from the Schedule 13D filings.

Panel A of Table 1 shows that Schedule 13D filers are large traders: the average (median) filer purchases 4.78\% (3.94\%) of outstanding shares during the 60-day disclosure period prior to the filing date. Figure 4 shows that these large traders become more aggressive as the event week approaches. Forty percent of Schedule 13D filers purchase shares seven weeks before the event week, and all of them purchase in the event week; they acquire 0.1\% of shares outstanding seven weeks before the event week, and they acquire 1.96\% of shares outstanding in the event week. These patterns are consistent with those in Brav et al. (2008) and Collin-Dufresne and Fos (2015).

We also find Schedule 13D filers are informed. In Figure 5, we plot the cumulative abnormal return (\textit{CAR}), relative to the Fama and French (1992) three-factor model, from the seventh week before the event week to the seventh week after the filing week. Specifically,

\[ CAR_i^{(k_1,k_2)} = \sum_{t=k_1}^{k_2} (r_{i,t} - \alpha_i - \sum_{m=1}^{3} \beta_{i,m} \cdot r_{m,t}), \]

where \( r_{i,t} \) is the log return of stock \( i \) on date \( t \), \( r_{m,t} \) is the log return of factor \( m \) (market, size, or book-to-market) on date \( t \), and \( \alpha_i \) and \( \beta_{i,m} \) are

\textsuperscript{11} Other filings are because (1) a group of investors’ acts as a legal group and the ownership of the group exceeds the 5\% reporting threshold, or (2) previously established positions change by more than 1\% of outstanding shares. We restrict our sample to original Schedule 13D filings where institutional activist shareholders exceed the 5\% threshold through open market purchases, because such events reflect the need of a single large informed trader.
estimated from the three-factor model using daily returns over a 260-trading day window ending nine weeks before the Schedule 13D filing date. \( CAR_{i}^{(k_{1}, k_{2})} \) cumulates the abnormal return from trading date \( k_{1} \) to trading date \( k_{2} \) around the filing date.\(^{12}\) As can be seen in Figure 5, there is a gradual run-up of about 2.5% over the seven-week period before the event week. The one-week jump in \( CAR \) is about 3.5% over the event week and 1.5% in the week before the filing week.\(^{13}\) Therefore, although the institutional activists are still the only informed traders in these periods, the price starts to move in the direction of their trades. The jump in \( CAR \) observed in the filing week is around 2.7%, when the intentions of the activists become public. After that the \( CAR \) continues to drift upwards for another two weeks and then remains at around 10%. In Table 1, Panel A shows that the average \( CAR \) for the \( (t - 1, t + 1) \), \( (t - 1, t + 11) \), and \( (t - 46, t + 11) \) trading-day windows around the Schedule 13D filing date \( t \) is 2.69%, 4.05%, and 9.62%, respectively.

5.2 Other Data Sources

The dependent variable we use to test predictions \( P1 \) and \( P2 \) is the market share of dark pools (\( DARKSHARE \)). Following SEC Concept Release (2010), we define alternative trading systems (ATSs) as dark pools, and collect the trading volume in ATSs for the universe of National Market System (NMS) securities from FINRA.\(^{14}\) The data start from May 2014, after FINRA Rule 4552 mandated that all ATSs have to report the weekly share volume executed in their systems. We aggregate the share volume traded in ATSs for each NMS security for each week to obtain firm-week observations of dark pool trading volume.\(^{15}\) We define \( DARKSHARE \) as share volume traded

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\(^{12}\) We obtain daily returns on Fama and French’s (1992) three factors from Kenneth French’s website at: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

\(^{13}\) There are 418 event weeks but only 137 weeks between event weeks and filing weeks in our sample.

\(^{14}\) TAQ data reports off-exchange trades, but off-exchange trades also include broker/dealer internalized retail trades. We believe FINRA ATS volume is a more accurate data source of dark pool trading volume than TAQ off-exchange volume.

\(^{15}\) To our best knowledge, there is no publically available dark pool trading volume data at daily or intraday frequency for our sample period.
in ATSs divided by total trading volume reported on the consolidated tap.

The dependent variable we use to test prediction $P3$ is price discovery. Following Weller (2017), we measure price discovery as the ratio of announcement $CAR$ to total $CAR$ before and including the announcement: $JUMP_i = \frac{CAR_i(-1, t+1)}{CAR_i(-46, t+11)}$. We define the announcement $CAR$ over the $(t - 1, t + 11)$ trading-day window due to the price drifts after the announcements of Schedule 13D filings shown in Figure 5. We define total $CAR$ over the $(t - 46, t + 11)$ trading-day window to also include the 60-calender-day disclosure period of Schedule 13D filings. A higher $JUMP$ ratio corresponds to a larger announcement price jump when holding the total amount of information constant, and thus represents less price discovery because less information is incorporated into prices before announcements.\(^{16}\) As pointed out in Weller (2017), the $JUMP$ ratio has an undefined conditional mean because its denominator may be close to zero. We resolve this problem by following his methodology to drop observations with small values of the denominator. Specifically, we retain events that satisfy $|CAR_i(-46, t+11)| > \sqrt{58} \cdot \hat{\sigma}_i$, where $\hat{\sigma}_i$ represents the standard deviation of the daily abnormal return over the 260-trading day estimation window. As reported in Table 1, Panel A, 182 out of 418 events remain after this filter, representing 44% of the events.\(^{17}\) The average $JUMP$ in our sample is 20.13%, suggesting that around 80% of Schedule 13D filers’ private information is reflected in prices before the filing date.

We construct other variables from standard datasets. Stock returns, total trading volume, and prices are from the Center for Research in Security Prices (CRSP). Intraday transactions data (trades and quotes) are from the daily Trade and Quote (TAQ) database. Balance sheet and income

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\(^{16}\) As we are measuring price discovery around a corporate event, this measure fits our purpose better than intraday price discovery measures, such as variance ratio or return autocorrelation at minute frequency.

\(^{17}\) The 44% survival rate is similar to the 45.5% rate reported in Weller (2017).
statement data are from Compustat. Analyst coverage data are from the Institutional Brokers’ Estimate System (I/B/E/S). Institutional ownership data are from Thomson Reuters’ 13f database.

5.3. Firm-week Observations and Their Matched Sample

For the 418 Schedule 13D filings, we retain the seven weeks before the event week, the event week, and the week between the event week and the filing week, if any. We restrict our analysis to firm-weeks that (1) have common stocks (CRSP share code 10 and 11); (2) have an average share price equal to or greater than $1; (3) have non-missing values for share volume in both CRSP and DTAQ; and (4) have non-missing values for both market capitalization at the end of the latest June and stock return volatility based on daily observations for the past year. The resulting sample includes 3,231 weeks for the target firms of Schedule 13D filings, 1,892 of which are with informed trades.

As activist institutional shareholders do not target firms randomly, we follow Collin-Dufresne and Fos (2015) to conduct difference-in-differences analysis on a matched sample of target and non-target firms. We match stocks based on propensity score, which is the predicted likelihood of being a target from a logistic regression model estimated on firm-years in the Compustat/CRSP universe. Following Brav et al. (2008, Table IV) and Collin-Dufresne and Fos (2015, Table IA.II), we regress a dummy variable of being targeted by any of the 418 Schedule 13D filings in our sample on lagged values of firm size, Tobin’s q, sales growth, return on assets, book leverage, dividend yield, R&D expenditures, Herfindahl index of segment sales, number of analysts following the firm, percentage of institutional ownership, firm age, annual stock returns, annual average of daily Amihud illiquidity, cash holdings, capital expenditures, annual average of weekly share turnover, and year dummies. We identify the matched stock as the non-target stock with the closest propensity score to the target firm in the same industry (three-digit SIC code.
industries) and year. For matched non-target firms, we collect the same calendar weeks as the matched target firms to create the matched sample of firm-weeks. The final matched sample includes a total of 6,416 observations for both target and non-target firms.

5.4 Summary Statistics

Panel B in Table 1 presents the summary statistics for the matched sample of firm-week observations. The mean weekly share turnover (TO) in the sample is 5.16%, among which 0.83% is executed in dark pools (TO_DARK) and 3.17% is executed on exchanges (TO_EXCH).\(^{18}\) D\(A\)R\(K\)SHARE, the percentage of total weekly trading volume executed in dark pools, is 15.34%, comparable to estimates reported in Kwan et al. (2015) and Menkveld et al. (2017).\(^{19}\) Scheduled 13D filers trade in roughly 58% of the weeks in the 60-calendar-day disclosure period, as indicated by the mean of dummy variable W13D. In the weeks when they trade, they purchase an average of $9,758,772 of target stocks or 0.84% of outstanding shares.

Panel C in Table 1 presents the comparison between weeks with and without Schedule 13D trades. Both the mean and median of TO_DARK and TO_EXCH are significantly higher during weeks with informed trades (13D weeks hereafter), relative to those without (non-13D weeks hereafter). The mean TO_DARK grows by 71% (1.19%/0.70% - 1), and mean TO_EXCH grows by 53% (4.27%/2.79% - 1). This finding is consistent with our model prediction that a large informed trader trades in both lit exchanges and dark pools, but more aggressively in the latter. As a result, DARKSHARE is significantly higher during 13D weeks than non-13D weeks. The increase is also economically significant: the mean (median) DARKSHARE increases by 2.07% (2.13%),

\(^{18}\) TO is larger than the sum of TO_DARK and TO_ON because a portion of the trading volume is executed in non-ATS OTC markets, which mainly consist of broker/dealer internalization.

\(^{19}\) Using proprietary data from NASDAQ for 2010-2011, Kwan et al. (2015) and Menkveld et al. (2017) estimate that the fraction of overall trading volume executed in crossing networks, dark LOBs, PING venues, and broker/dealer internalization is around 2.5%, between 7.8% and 9.8%, between 4.5% and 7.8%, and between 5.7% and 12.9%, respectively.
representing 14.21% (14.84%) of the level in non-13D weeks. We also find 13D weeks have significantly higher total share turnover (TO), trade size (TRADSIZE), and depth (DEP), and significantly lower quoted spread (QSPREAD) and intraday return volatility (RETVAR), consistent with Collin-Dufresne and Fos (2015).

Panel C also presents the comparison between 13D weeks of target firms and the same calendar weeks of matched firms. The means and medians of DARKSHARE, TO_DARK, and TO_EXCH are significantly higher for the target firms than the matched firms. The mean (median) of DARKSHARE is 1.53% (1.60%) higher, representing 10% (11%) of the level of matched stocks.

6. Results

This section presents the results of testing the predictions we discuss in Section 4.

6.1. Dark Pool Make Share

We first examine the relation between dark pool market share and Schedule 13D informed trades. In Figure 6, we present the average share turnover in dark pools (TO_DARK), average share turnover on exchanges (TO_EXCH), and average dark pool market share (DARKSHARE) from the seventh week before the event week to the seventh week after the filing week for the sample of 427 Schedule 13D filings. To remove secular trends, we plot market-adjusted value instead of the raw value in Figure 3. Figure 6 (a) and (b) show that TO_DARK and TO_EXCH increase gradually as the event week approaches, jump up in the event week, and then decrease after the filing week. The surge of trading volume in the even week is partly due to the intensive purchases by Schedule 13D filers. Turning to Figure 6 (c), market-adjusted DARKSHARE increases gradually from around 0% to around 1.5% in the event week, peaks at 2% in the week before the filing week, and then decreases gradually to around 0% after the filing week. This pattern is consistent with the
interpretation that large informed traders trade more aggressively in dark pools when exploiting private information.

Next, we test our prediction P1 that dark pool market share increases with informed trades following the difference-in-differences approach in Collin-Dufresne and Fos (2015). This methodology removes secular fluctuations in dark pool market share between weeks with and without informed trades and parallel trends of dark pool market share between target and non-target firms with similar characteristics. Specifically, we estimate the following regression:

\[ \text{DARKSHARE}_{i,t} = \alpha + W13D_{i,t} + \beta_2 W13D_{i,t} \ast \text{TARGET}_i + \sum \text{CONTROL}_{i,t} + \gamma_i + \epsilon_{i,t}, \]  

(22)

where \( \text{TARGET}_i \) is an indicator variable equal to one for firms targeted by activists in Schedule 13D filings and zero for the propensity-score matched non-target firms. For each target firm (\( \text{TARGET}_i = 1 \)), the indicator variable \( W13D_{i,t} \) equals to one for weeks with Schedule 13D trades and zero otherwise. For each non-target firm, \( W13D_{i,t} \) is assigned the same value of the same calendar week for the matched target firm. The control variables in equation (22) include a list of liquidity variables (\( \text{TO}, \text{TRADESIZE}, \text{QSPREAD}, \text{DEP}, \text{ABSOIMB}, \) and \( \text{RETVAR} \)) and a list of firm characteristics (\( \text{PRC}, \text{MV}, \) and \( \text{VOLATILITY} \)) that have been shown to explain dark pool market share (e.g., O’Hara and Ye, 2011). All variables are defined in Appendix B. The main effect of \( \text{TARGET}_i \) is absorbed by event fixed effects \( \gamma_i \), and we are interested in the coefficient before the interaction term \( \beta_2 \).

Column (1) of Table 2 shows that \( \text{DARKSHARE} \) increases by 1.9% (\( p < .01 \)) in 13D weeks relative to non-13D weeks, consistent with our prediction P1. This increase represents 13% (=1.9%/14.57%) of the level in the non-13D weeks. Next, we explore the variation in quantity of informed trades across the 13D weeks of target firms (\( W13D_{i,t} \ast \text{TARGET}_i = 1 \)) by replacing \( W13D_{i,t} \ast \text{TARGET}_i \) with the natural logarithm of the dollar amount of net purchases of the target
stock \( i \) over 13D week \( t \) by the 13D filer \( (BUY13D$) \). Column (2) presents the results. The coefficient on \( LN(BUY13D$) \) is significantly positive, indicating that \( DAKRSHARE \) increases more when Schedule 13D filers purchase more in the target firms.

While an increase in \( DAKRSHARE \) during 13D weeks is consistent with our first prediction, it is subject to alternative explanations. One potential alternative explanation is that activists time their trades when the market is more liquid (Collin-Dufresne and Fos, 2015) and dark pool market share is higher when liquidity is higher (Buti et al., 2017). As a result, even if institutional activists did not trade in dark pools, we would still observe an increase in dark pool market share during 13D weeks. We rule out this alternative explanation in three ways. First, we identify two events in our sample for which Schedule 13D filers disclose trades at the transaction level. We find a large fraction of transactions executed at sub-penny prices, which are mostly trades in dark pools (Zhu, 2014). This finding suggests that institutional activists do trade in dark pools. Second, we control for a list of liquidity measures (e.g., total share turnover, bid-ask spread, and intraday volatility) in the regressions for Table 2, and the results suggest that the increase in \( DAKRSHARE \) during 13D weeks is incremental to the effect of liquidity on \( DAKRSHARE \). Finally, we re-estimate the regression for column (2) by replacing the dollar amount of net purchases \( (BUY13D$) \) with the ratio of net shares purchased by Schedule 13D filers to total share volume over the 13D week \( (BUY13D\_RATIO) \). If institutional activists simply execute more shares on exchanges when overall liquidity is higher, we would not observe a relation between \( BUY13D\_RATIO \) and \( DAKRSHARE \). However, if they trade more aggressively in dark pools to hide their private information, we would observe a positive relation between \( BUY13D\_RATIO \) and \( DAKRSHARE \). Inconsistent with the alternative explanation but consistent with our model, we find a significantly

\[ \text{In untabulated analyses, we confirm that the relation between liquidity and } DAKRSHARE \text{ does not change during 13D weeks.} \]
positive coefficient of 0.039 ($p<.01$) on $BUY13D\_RATIO$ in column (3). Overall, we find evidence supporting our first prediction P1 that dark pool market share increases when a large informed trader aggressively acquires shares of a firm.

Turning to our second prediction, we examine the variation in the increase of dark pool market share over 13D weeks with the value of private information and the historical market share of the dark pool, both of which are measured at the filing level. We use $CAR$ over the $(t – 46, t + 11)$ trading-day window around the filing date ($INFOR$) to measure the value of private information possessed by the Schedule 13D filer.\(^{21}\) The historical market share of dark pool ($HIS\_DARKSHARE$) is the market-adjusted dark pool market share of the target firm over the year prior to the 60-calendar-day disclosure period. We use market-adjusted value instead of raw value to calculate $HIS\_DARKSHARE$ as both dark pool market share and the likelihood of Schedule 13D filing may vary together with market conditions.

To test prediction P2, we replace $TREAT_i$ in equation (22) with $INFOR_i$ or $HIS\_DARKSHARE_i$. For non-target firms (i.e., $TREAT_i = 0$), we code $INFOR_i$ and $HIS\_DARKSHARE_i$ as 0. Column (1) of Table 3 presents the results for the regression where we replace $W13D_{i,t}*TARGET_i$ with $W13D_{i,t}*INFOR_i$ in equation (22). We find a significantly positive coefficient of 0.024 ($p<.05$) for $W13D_{i,t}*INFOR_i$, suggesting that the increase in the dark pool market share during 13D weeks is larger when Schedule 13D filers possess more valuable private information and therefore have a stronger incentive to hide their information in dark pools.

Column (2) of Table 3 presents results for the regression in which we replace $W13D_{i,t}*TARGET_i$ with $W13D_{i,t}*HIS\_DARKSHARE_i$ in equation (22). We find a significantly

\(^{21}\) $INFOR$ includes returns over the 60 days prior to the public announcement as part of the private information of the Schedule 13D filer and gets incorporated into the stock price when she trades. $INFOR$ includes returns in the two-week period after the filing date as stock price continues to increase, as shown in Figure 2.
positive coefficient 0.115 ($p<.01$) for $W13D_{i,t} \times HIS_{\text{DARKSHARE}_i}$, suggesting that an increase in dark pool market share during 13D weeks is larger when the historical market share of the dark pool is higher. This result suggests that Schedule 13D filers are more likely to trade in dark pools relative to exchanges when the dark pool is more liquid historically.

We next include $W13D_{i,t} \times INFOR_i$ and $W13D_{i,t} \times HIS_{\text{DARKSHARE}_i}$ in the same regression and report the results in column (3). The results in columns (1) and (2) remain largely unchanged. In terms of economic significance, a one standard deviation (30.03%) increase in $INFOR_i$ is associated with a 0.84% (= 0.028*30.03%) higher $DARKSHARE$ and a one standard deviation (4.61%) increase in $HIS_{\text{DARKSHARE}_i}$ is associated with a 0.48% (= 0.104*4.61%) higher $DARKSHARE$ during 13D weeks relative to non-13D weeks. Overall, the evidence supports our prediction P2 that the increase of dark pool market share during times of informed trades is larger when the value of private information is larger and when relative liquidity in the dark pool is higher.

6.2. Price Discovery

We next examine the variation in price discovery ($JUMP$) as it relates to the value of private information possessed by Schedule 13D filers ($INFOR$) and the historical market share of dark pools ($HIS_{\text{DARKSHARE}}$). We estimate the following regression on the sample of 182 Schedule 13D events with non-missing $JUMP$:

$$JUMP_i = \alpha + \beta_1 \cdot INFOR_i + \beta_2 \cdot HIS_{\text{DARKSHARE}_i} + \sum CONTROL_i + \varepsilon_i.$$  \hspace{1cm} (23)

Under prediction P3, we expect positive $\beta_1$ and $\beta_2$. In column (1) of Table 4, we find a significantly positive coefficient of 0.119 ($p<.10$) on $INFOR_i$, consistent with our prediction that price discovery is weaker when the informed trader possesses more valuable information and thus has a stronger incentive to hide her information in dark pools to minimize information leakage. In column (2), we find a significantly positive coefficient of 1.724 ($p < 0.10$) for $HIS_{\text{DARKSHARE}_i}$,
consistent with our prediction that price discovery is weaker when historical dark pool liquidity is higher as Schedule 13D filers can trade more in dark pools to minimize price impact. The regression for column (3) includes both $INFOR_i$ and $HIS DARKSHARE_i$. Conclusions drawn from columns (1) and (2) remain nearly identical. In terms of economic magnitude, we find a one standard deviation increase in $INFOR_i$ (42.91%, untabulated) is associated with a 7.72% (=0.180*42.91%) higher $JUMP$ ratio and a one standard deviation increase in $HIS DARKSHARE_i$ (4.08%, untabulated) is associated with a 7.30% (=1.788*4.08%) higher $JUMP$ ratio, both are significant when compared with the mean $JUMP$ ratio of 20.13%. Overall, the evidence supports prediction P3 that price discovery decreases with the value of private information and the relative liquidity in the dark pool.

7. Conclusion

In this paper, we link the literature on corporate governance with the literature on market microstructure. To intervene in corporate governance, activist institutional shareholders need to acquire a significant stake in a firm. Yet there is a paucity of information on where they acquire those shares. Our paper adds to the literature by demonstrating the importance of dark pools in this process.

To guide the empirical analysis on the venue choice of activist institutional investors, we first construct a parsimonious model to show the economic mechanisms that drive the importance of the dark pool. The model features the order-splitting behavior of a large informed trader. We find that the large informed trader trades in both the lit exchange and the dark pool, but she trades relatively more in the dark pool than in the exchange to hide her information. Therefore, the informed trader increases the market share of the dark pool. The increase in dark pool market share
is larger, thus price discovery is lower when the private information possessed by the informed trader is more valuable and when the relative liquidity trading in the dark pool is higher. Using a comprehensive sample of Schedule 13D filings and FINRA ATS volume data, we provide evidence that is consistent with our model predictions.

Our paper can be extended both theoretically and empirically. In terms of theory, we develop the quadratic power-law framework and use it to solve the first rational expectation equilibrium model with both price and execution uncertainty, but our model is parsimonious. For example, in our model, we assume that there are exogenous liquidity traders, whereas in a more complex analysis they can be endogenous. Allowing liquidity traders to choose the size and venue based on their hedging motive (Spiegel and Subrahmanyam, 1992) or liquidity preference (Mendelson and Tunca, 2004) enables the analysis of the welfare of traders, which is an interesting and important question. In terms of empirics, our results are suggestive because U.S. data do not allow us to see the actual orders of activist institutional shareholders. Data that contain more refined information, probably from another country, may provide more information on how and where these activists acquire large stakes of stocks.

Finally, our findings indicate that activists extensively use dark pools. One major conjecture is that reform and the evolution of market structure may affect shareholder activism. It will be fruitful to test this prediction using exogenous shocks on market structure.
References

Collin-Dufresne, P., and V. Fos. 2015. Do Prices Reveal the Presence of Informed Trading?


Appendix A. Proofs

Proof of Lemma 1

Let $G(x_d) = E(\bar{x}_m|x_d)$. Then:

$$
G'(x_d) = x_d f_s(x_d) - x_d f_s(x_d) + \int_{x_d}^{+\infty} f_s(z)dz = \int_{x_d}^{+\infty} f_s(z)dz > 0. \quad (A.1)
$$

$$
\left(\frac{G(x_d)}{x_d}\right)' = \frac{G'(x_d)x_d - G(x_d)}{x_d^2} = \frac{x_d \int_{x_d}^{+\infty} f_s(z)dz - \int_{x_d}^{+\infty} z f_s(z)dz - x_d \int_{x_d}^{+\infty} f_s(z)dz + \int_{x_d}^{+\infty} z f_s(z)dz}{x_d^2} = -\frac{\int_{x_d}^{+\infty} z f_s(z)dz}{x_d^2} < 0. \quad (A.2)
$$

Proof of Lemma 2

The expected volume for any $x_d$ is:

$$
E(\bar{x}_m|x_d) = \int_0^{x_d} z f_s(z)dz + x_d \int_{x_d}^{+\infty} f_s(z)dz = 2k\frac{1}{2}(x_d + k)^{\frac{1}{2}} - 2k. \quad (A.3)
$$

Substituting the expression of $x_d^*$ into (A.3), we obtain:

$$
E(\bar{x}_m|x_d^*) = 2k\frac{1}{c} (v_g - \mu - \lambda x_e) - 2k = 2k\frac{1}{c} (v - \mu - \lambda x_e). \quad (A.4)
$$

Therefore,

$$
E(\pi_d^*) = E(\bar{\pi})E(\bar{x}_m|x_d^*) - x_d^* \cdot c
$$

$$
= \left( v_g - \mu - \lambda x_e \right) \frac{2k}{c} (v - \mu - \lambda x_e) - \left( \frac{k}{c^2} \left( v_g - \mu - \lambda x_e \right)^2 - k \right) c
$$

$$
= \frac{k}{c} (v - \mu - \lambda x_e)^2. \quad (A.5)
$$

Proof of Theorem 1

Define function $y = f(\lambda) = \frac{(1 - 2\lambda k)}{(2\lambda - 2\lambda^2 k)^2}$. 

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\( f(\lambda) \) is continuous on \((-\infty, 0) \cup \left(0, \frac{1}{K}\right) \cup \left(\frac{1}{K}, +\infty\right). \) And we have:

\[
 f'(\lambda) = \left(\frac{-2K(2\lambda - 2\lambda^2 K)^2 - (1 - 2\lambda K)^2 + (1 - 2\lambda^2 K)(2 - 4\lambda K)}{(2\lambda - 2\lambda^2 K)^4}\right). \tag{A.6}
\]

The denominator of (A.6) is equal to 0 when \( \lambda = 0 \) or \( \lambda = \frac{1}{K} \), and greater than 0 otherwise. The numerator can be simplified as \( 24\lambda(\lambda K - 1) \left[\left(\frac{1}{2}\right)^2 + \frac{1}{12}\right] \).

So, it is easy to see:

\[
\begin{cases} 
 f'(\lambda) > 0 \text{ when } \lambda < 0 \\
 f'(\lambda) < 0 \text{ when } 0 < \lambda < \left(\frac{1}{K}\right) \\
 f'(\lambda) > 0 \text{ when } \lambda > \left(\frac{1}{K}\right)
\end{cases}
\]

Also, \( \lim_{\lambda \to -\infty} f(\lambda) = 0, \lim_{\lambda \to 0^+} f(\lambda) = +\infty, \lim_{\lambda \to \left(\frac{1}{K}\right)^-} f(\lambda) = +\infty, \lim_{\lambda \to \left(\frac{1}{K}\right)^+} f(\lambda) = -\infty, \)

\[ \lim_{\lambda \to \left(\frac{1}{K}\right)^+} f(\lambda) = -\infty, \lim_{\lambda \to +\infty} f(\lambda) = 0 \text{ and } f\left(\frac{1}{2K}\right) = 0. \]

So \( y = f(\lambda) = R \ (R > 0) \) has two real solutions,

\[
\lambda_1^* \in \left(0, \frac{1}{2K}\right) \text{ and } \lambda_2^* \in (-\infty, 0). \tag{A.7}
\]

From (14), \( \beta = \frac{1 - 2\lambda K}{2\lambda - 2\lambda^2 K} = \frac{1 - 2\lambda K}{2\lambda(1 - \lambda K)}. \) Therefore:

\[
\lambda_1^* \in \left(0, \frac{1}{2K}\right) \Rightarrow \beta_1^* > 0 \text{ and } \lambda_2^* \in (-\infty, 0) \Rightarrow \beta_2^* < 0. \tag{A.8}
\]

Denote \( \pi = E(\pi_f + \tilde{\pi}_f) \). The Hessian of \( \pi \) is:

\[
\begin{bmatrix}
\frac{\partial^2 \pi}{\partial x_e^2} & \frac{\partial^2 \pi}{\partial x_e \partial x_d} \\
\frac{\partial^2 \pi}{\partial x_e \partial x_d} & \frac{\partial^2 \pi}{\partial x_d^2}
\end{bmatrix} = \begin{bmatrix}
-2\lambda & -\lambda \int_{x_d}^{+\infty} f_s(z) \, dz \\
-\lambda \int_{x_d}^{+\infty} f_s(z) \, dz & -(\nu_d - \mu - \lambda x_e) f_s(x_d)
\end{bmatrix}.
\]
For $\lambda_2^* \in (-\infty, 0)$, the first-order principle minor is positive. Therefore, the Hessian matrix with $\lambda_2^*$ cannot be negative semidefinite, which violates the necessary condition for profit maximization.

When $\lambda_1^* \in \left(0, \frac{1}{2k}\right)$, the first-order principle minor is negative. Now we need to show that the second-order principle minor is positive, that is:

$$-2\lambda \left(-(v_g - \mu - \lambda x_e) f_s(x_d^*)\right) - \left(-\lambda \int_{x_d}^{+\infty} f_s(z)dz\right)^2 > 0. \quad (A.9)$$

Combining (2) and (11) yields:

$$f_s(x_d^*) = \frac{1}{2^3} k^3 \left(v_g - \mu - \lambda x_e\right)^2 - k + k \right)\left(v_g - \mu - \lambda x_e\right)^{-3}. \quad (A.10)$$

Plugging (A.10) into (A.9):

$$-2\lambda \left(-(v_g - \mu - \lambda x_e) f_s(x_d^*)\right) - \left(-\lambda \int_{x_d}^{+\infty} f_s(z)dz\right)^2 \right)$$

$$= \lambda \frac{c^3}{k} (v_g - \mu - \lambda x_e)^{-2} \left(1 - \left(\frac{k}{c}\right)\lambda\right). \quad (A.11)$$

Because $\lambda \in \left(0, \frac{1}{2k}\right)$ and $\frac{k}{c} \equiv K, \left(1 - \frac{k}{c}\lambda\right) > 0$. Also $\lambda > 0$ and $\frac{c^3}{k} (v_g - \mu - \lambda x_e)^{-2} > 0$. Therefore, the second principle minor for $\lambda_1^* \in \left(0, \frac{1}{2k}\right)$ is greater than 0. So, Hessian for $\lambda_1^*$ is negative definite, which is the sufficient condition for profit maximization. ■

**Proof of Corollary 1**

The comparative statics of $\lambda^*$ follow from the implicit function rule. Equations (14) and (21) define two implicit functions $\lambda^*(R, K)$ and $\beta^*(R, K)$, where $R \equiv \frac{\sigma^2}{\sigma^0}$ and $K \equiv \frac{k}{c}$. Fix $c$, the sign of $\frac{\partial \lambda^*}{\partial K}$ is the same as $\frac{\partial \lambda^*}{\partial K}$, and the sign of $\frac{\partial \beta^*}{\partial K}$ is the same as $\frac{\partial \beta^*}{\partial K}$.\[46\]
Denote:
\[
\begin{align*}
F^1(\lambda^*, \beta^*; R, K) &= (2\lambda^* - 2\lambda^{*2}K)\beta^* - (1 - 2\lambda^* K) = 0 \\
F^2(\lambda^*, \beta^*; R, K) &= R(2\lambda^* - 2\lambda^{*2}K)^2 - (1 - 2\lambda^* K) = 0
\end{align*}
\] (A.12)

Taking total derivatives with respect to \( R \) and \( K \), we get:
\[
\begin{bmatrix}
(2\lambda^* - 2\lambda^{*2}K) \\
0
\end{bmatrix} (2 - 4\lambda^* K)\beta + 2K \\
\begin{bmatrix}
\frac{\partial \beta^*}{\partial R} & \frac{\partial \beta^*}{\partial K}
\end{bmatrix}
\begin{bmatrix}
2R(2\lambda^* - 2\lambda^{*2}K)(2 - 4\lambda^* K) + 2K \\
\frac{\partial \lambda^*}{\partial R} & \frac{\partial \lambda^*}{\partial K}
\end{bmatrix}
= \begin{bmatrix}
0 \\
-(2\lambda^* - 2\lambda^{*2}K)^2
\end{bmatrix} 2R(2\lambda^* - 2\lambda^{*2}K)2\lambda^* - 2\lambda^* \\
\begin{bmatrix}
2\lambda^2\beta - 2\lambda^* \\
2R(2\lambda^* - 2\lambda^{*2}K)2\lambda^* - 2\lambda^*
\end{bmatrix}
\] (A.13)

\[
\begin{align*}
(\text{A.13}) \Rightarrow \frac{\partial \lambda^*}{\partial R} &= \frac{(2\lambda^* - 2\lambda^{*2}K)^2}{2R(4\lambda^* K - 2)(2\lambda^* - 2\lambda^{*2}K) - 2K}.
\end{align*}
\] (A.14)

For \( K > 0 \), \( R > 0 \) and \( \lambda^* \in \left(0, \frac{1}{2K}\right) \) \( \Rightarrow \begin{cases} 
2R(4K\lambda^* - 2) < 0 \\
2\lambda^* - 2\lambda^{*2}K > 0 \\
-2K < 0
\end{cases} \) \Rightarrow 2R(4K\lambda^* - 2)(2\lambda^* - 2\lambda^{*2}K) - 2K < 0 \Rightarrow \frac{\partial \lambda^*}{\partial R} < 0 \] (A.15)

\[
(\text{A.13}) \Rightarrow \frac{\partial \lambda^*}{\partial K} = \frac{2R(2\lambda^* - 2\lambda^{*2}K)(2\lambda^{*2} - 2\lambda^*)}{2R(2\lambda^* - 2\lambda^{*2}K)(2 - 4\lambda^* K) + 2K}.
\] (A.16)

Plug equation (21) to (A.16) to eliminate \( R \):
\[
\frac{\partial \lambda^*}{\partial K} = \frac{-\lambda^{*3}K}{3\lambda^{*2}K^2 - 3\lambda^* K + 1} = -\frac{\lambda^{*3}K}{3\left(\lambda^* K - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} < 0.
\] (A.17)

From the proof of Theorem 1, we know that \( \lambda^* \) is bounded by \( \left(0, \frac{1}{2K}\right) \), when \( K \to +\infty, \frac{1}{2K} \to 0, \lambda^* \to 0 \). □

**Proof of Corollary 2**

The proof of Corollary 2 is divided into three parts. First, we prove that both \( \beta^* \) and \( x_d^* \) are positive unless \( \nu = p_0 \). Then we show the comparative statics of \( \beta^* \). Finally, we show the comparative statics of \( x_d^* \).
\[ \beta^* > 0 \text{ follows from Corollary 1. For } x_d^*, \text{ we focus the proof on the case when } v > p_0, \text{ and the case for } v < p_0 \text{ is simply symmetric. When } v > p_0, \]

\[ x_d^* = \frac{k}{c^2} (v + c - p_0 - \lambda^* x_e)^2 - k = \frac{k}{c^2} (v - p_0 - \lambda^* \beta^*(v - p_0) + c)^2 - k \]

\[ = \frac{k}{c^2} ((v - p_0)(1 - \lambda^* \beta^*) + c)^2 - k. \tag{A.18} \]

From equation (17), \[\lambda^* \beta^* = \frac{\beta^2}{\beta^2 + R} < 1.\] Therefore, \[(v - p_0)(1 - \lambda^* \beta^*) + c > c,\] so

\[ x_d^* > \frac{k}{c^2} (c)^2 - k = 0. \tag{A.19} \]

With definitions of \(F_1^\prime\) and \(F_2^\prime\) and (A.13) from Proof of Corollary 1, we have:

\[ (A.13) \Rightarrow \frac{\partial \beta^*}{\partial R} = \frac{(4 \lambda^* K - 2) \beta^* - 2 K \lambda^*}{(2 \lambda^* - 2 \lambda^2 K)} \frac{\partial \lambda^*}{\partial R} \tag{A.20} \]

Plug (14) into (A.20) to eliminate \(\beta^*\):

\[ \frac{\partial \beta^*}{\partial R} = \frac{-(1 - 2 \lambda^* K)^2}{2 \lambda^* - 2 \lambda^2 K} \frac{2 \lambda^* - 2 \lambda^2 K}{(2 \lambda^* - 2 \lambda^2 K)} \frac{\partial \lambda^*}{\partial R} > 0. \tag{A.21} \]

Because \(K > 0, 2 \lambda^* - 2 \lambda^2 K > 0, -(1 - 2 \lambda^* K)^2 < 0\) and \(\frac{\partial \lambda^*}{\partial R} < 0.\)

\[ (A.13) \Rightarrow \frac{\partial \beta^*}{\partial K} = \frac{2 \lambda^2 \beta^* - 2 \lambda^* + [(4 \lambda^* K - 2) \beta^* - 2 K] \left(\frac{\partial \lambda^*}{\partial K}\right)}{(2 \lambda^* - 2 \lambda^2 K)}. \tag{A.22} \]

Plug in the expression of \(\beta^*\) and \(\left(\frac{\partial \lambda^*}{\partial K}\right)\):

\[ \frac{\partial \beta^*}{\partial K} = \frac{-2 \lambda^2}{(2 \lambda^* - 2 \lambda^2 K)^2} + \frac{4 \lambda^5 K^3 - 4 \lambda^4 K^2 + 2 \lambda^3 K}{(2 \lambda^* - 2 \lambda^2 K)^2 (3 \lambda^2 K^2 - 3 \lambda^* K + 1)} \]

\[ = \frac{2 \lambda^* K - 1}{2(3 \lambda^2 K^2 - 3 \lambda^* K + 1)} < 0. \tag{A.23} \]
For $\frac{\partial x^*_d}{\partial R}$, we focus on the case when $v > p_0$. The case when $v < p_0$ can be proved symmetrically.

$$\frac{\partial x^*_d}{\partial R} = \frac{k}{c^2} 2((1 - \lambda^* \beta^*) (v - p_0) + c) \left[ (v - p^0) \left(-\lambda^* \frac{\partial \beta^*}{\partial R} - \beta^* \frac{\partial \lambda^*}{\partial R} \right) \right]. \quad (A.25)$$

As $\lambda^* \beta^* = \frac{\beta^2}{\beta^{*2 + R}} < 1$ and $v - p_0 > 1$, the sign of $\frac{\partial x^*_d}{\partial R}$ is determined by $\left(-\lambda^* \frac{\partial \beta^*}{\partial R} - \beta^* \frac{\partial \lambda^*}{\partial R}\right)$. However,

$$\left(-\lambda^* \frac{\partial \beta^*}{\partial R} - \beta^* \frac{\partial \lambda^*}{\partial R}\right) = \frac{2K(1 - \lambda^* \beta^*) \partial \lambda^*}{2 - 2\lambda^* K} < 0. \quad (A.26)$$

Because $\lambda^* \beta^* = \frac{\beta^2}{\beta^{*2 + R}} < 1$, $\lambda^* \in \left(0, \frac{1}{2K}\right)$ and $\frac{\partial \lambda^*}{\partial R} < 0$. $\frac{\partial x^*_d}{\partial R} < 0$.

To know the sign of $\frac{\partial x^*_d}{\partial K}$, we only need to know the sign of $\frac{\partial x^*_d}{\partial K}$ when $c$ is fixed:

$$\frac{\partial x^*_d}{\partial K} = \frac{1}{c} (1 - \lambda^* \beta^*) (v - p_0) + c)^2 - c$$

$$+ \frac{K}{c} ((1 - \lambda^* \beta^*) (v - p_0) + c) \left(-\frac{\partial \lambda^*}{\partial K} \beta^* - \lambda^* \frac{\partial \beta^*}{\partial K}\right). \quad (A.27)$$

Note that $\frac{1}{c} ((1 - \lambda^* \beta^*) (v - p_0) + c)^2 - c = \frac{1}{c} \left[ ((1 - \lambda^* \beta^*) (v - p_0) + c)^2 - c^2 \right] = \frac{1}{c} \left[ ((1 - \lambda^* \beta^*) (v - p_0))((1 - \lambda^* \beta^*) (v - p_0) + 2c) \right] > 0.$

$$\frac{K}{c} 2 ((1 - \lambda^* \beta^*) (v - p_0) + c) \left(-\frac{\partial \lambda^*}{\partial K} \beta^* - \lambda^* \frac{\partial \beta^*}{\partial K}\right) > 0 \text{ because } ((1 - \lambda^* \beta^*) (v - p_0) + c) > 0, \frac{\partial \lambda^*}{\partial K} < 0,$$

$\frac{\partial \beta^*}{\partial K} < 0, \beta^* > 0$ and $\lambda^* > 0$. Therefore, $\frac{\partial x^*_d}{\partial K} > 0.$

**Proof of Theorem 2**

$$E(\bar{x}_m|x^*_d) = \frac{2k}{c} (v - p_0 - \lambda^* x^*_d) x^*_e = \frac{2K(v - p_0 - \lambda^* \beta^* (v - p_0))}{\beta^* (v - p_0)} = \frac{2K(1 - \lambda^* \beta^*)}{\beta^*}. \quad (A.28)$$

Plugging (14) into equation (A.28), we obtain:
\[
\frac{E(\bar{x}_m|x_d)}{x_e} = 2K \frac{\left(1 - \lambda^* \left(\frac{1 - 2\lambda^* K}{2\lambda^* - 2\lambda^{*2} K}\right)\right)}{1 - 2\lambda^* K} = \frac{2K(\lambda^*)}{1 - 2\lambda^* K} \frac{1}{1 - 2\lambda^* K} = 1 - \frac{1}{1 - 2\lambda^* K}. \quad (A.29)
\]

Denote \(2 - 2\lambda^* K \equiv l\), then:

\[
\lambda^* = \frac{2 - l}{2K}. \quad (A.30)
\]

Plugging (A.30) into (21) yields:

\[
\frac{(2 - l)^2 l^2}{(l - 1)} = 4 \left(\frac{k}{\sigma_v c}\right)^2. \quad (A.31)
\]

Surely, the solution of \(l\) only depends on \(\left(\frac{k}{\sigma_v c}\right)^2\), the unique parameter in (A.31).

Following similar steps to proving Corollary 1, we can show there are two real solutions to this model:

\[
l^*_1 \in (1,2) \text{ and } l^*_2 \in (2, +\infty). \quad (A.32)
\]

However, \(l^*_2\) is ruled out because it implies \(\lambda^*_2 \in (-\infty, 0)\). Therefore, we get optimal \(l^* = l^*_1\), which is uniquely determined by \(\left(\frac{k}{\sigma_v c}\right)^2\).

Denote \(\left(\frac{k}{\sigma_v c}\right)^2 = N\). Then (A.31) becomes:

\[
(2 - l)^2 l^2 = 4N (l - 1). \quad (A.33)
\]

Totally differentiating both sides of the equation and evaluating the equation at \(l^*\) yields:

\[
\frac{dl^*}{dN} = \frac{l^* - 1}{l^*(l^* - 1)(l^* - 2) - N} < 0 \quad (A.34)
\]

because \(N > 0, l^* \in (1,2)\).
Therefore, \( l \equiv 2 - 2\lambda^*K \) decreases in \( \frac{k\sigma_v}{\sigma_e^c}^2 \). \( l - 1 \equiv 1 - 2\lambda^*K \) is also uniquely determined by \( \frac{k\sigma_v}{\sigma_e^c}^2 \) and decreases in \( \frac{k\sigma_v}{\sigma_e^c}^2 \). So \( \frac{\beta(X_m|X_d)}{x_e} = \frac{1}{1 - 2\lambda^*K} - 1 \) is uniquely determined by \( \frac{k\sigma_v}{\sigma_e^c}^2 \) and increases in \( \frac{k\sigma_v}{\sigma_e^c}^2 \).

\[ \text{∎} \]

**Proof of Theorem 3**

The market maker sets the price based on \( y = \beta^*(\bar{v} - p_0) + \bar{u}_e \). Rearranging terms yields:

\[ \theta \equiv \frac{y}{\beta^*} + p_0 = \bar{v} + \frac{\bar{u}_e}{\beta^*} \quad (A.35) \]

\( \theta \) is an informationally equivalent transformation of the observed order flow \( y \). In addition, \( \theta \) has the same mean as \( \bar{v} \). The distribution of \( \theta \) conditional on \( v \) is \( \theta|v \sim N(v, \frac{\sigma_v^2}{\beta^*}) \). The definition of price informativeness follows Kyle (1985), which is equal to 1 minus the ratio of the posterior variance of \( \bar{v} \) to the prior variance of \( \bar{v} \), denoted:

\[ e = 1 - \frac{\text{var}(\bar{v}|\bar{p})}{\text{var}(\bar{v})} \quad (A.36). \]

Note that \( \text{var}(\bar{v}|\bar{p}) = \text{var}(\bar{v}|\bar{y}) = \text{var}(\bar{v}|\bar{\theta}) \) because \( \bar{\theta}, \bar{p}, \) and \( \bar{y} \) are informationally equivalent. Bayes’ rule implies that the posterior variance \( \text{var}(\bar{v}|\bar{p}) \) is:

\[ \text{var}(\bar{v}|\bar{p}) = \text{var}(\bar{v}|\bar{\theta}) = \left( \frac{1}{\sigma_v^2} + \frac{\beta^*}{\sigma_e^2} \right)^{-1} \cdot \frac{\sigma_v^2}{1 + \beta^* \frac{\sigma_v^2}{\sigma_e^2}} \quad (A.37). \]

Plugging equations (14) and (21) into equation (A.37):

\[ e = 1 - \frac{1}{1 + \frac{\beta^* \sigma_v^2}{\sigma_e^2}} = 1 - \frac{1}{1 + R(1 - 2\lambda^*K) \frac{1}{R}} = 1 - \frac{1}{2 - 2\lambda^*K} \quad (A.38). \]

From equation (A.38), \( e \) is uniquely determined by \( 2 - 2\lambda^*K \). From the proof of Theorem 2, we have \( l \equiv 2 - 2\lambda^*K \) and show it is uniquely determined by \( \frac{k\sigma_v}{\sigma_e^c}^2 \) and decreases in \( \frac{k\sigma_v}{\sigma_e^c}^2 \). It is then easy to verify that \( e = 1 - \frac{1}{2 - 2\lambda^*K} \) decreases in \( \frac{k\sigma_v}{\sigma_e^c}^2 \). \( \text{∎} \)
## Appendix B
### Variable Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variables of Interest</strong></td>
<td></td>
</tr>
<tr>
<td><strong>BUY13D</strong></td>
<td>Net dollar amount ($) or net percentage ownership (%) of the target firm purchased by the Schedule 13D filer during a week.</td>
</tr>
<tr>
<td><strong>BUY13D_RATIO</strong></td>
<td>The ratio of net shares purchased by the Schedule 13D filer over a week to total share volume reported in CRSP.</td>
</tr>
<tr>
<td><strong>CAR (-k₁, +k₂)</strong></td>
<td>Cumulative abnormal return over the ((t - k₁, t + k₂)) trading-day window around the filing date (t) of Schedule 13D. Abnormal return is (r_{i,d} - \alpha_i - \sum_{m=1}^{3} \beta_{i,m} \cdot r_{m,d}), where (r_{i,d}) is the log return of stock (i) on date (d), (r_{m,d}) is the log return of factor (m) (market, size, or book-to-market) on date (d), and (\alpha_i) and (\beta_{i,m}) are estimated from the Fama and French (1992) three factor model using daily returns over a 260-trading day window ending 9 weeks before the Schedule 13D filing date. Daily returns on Fama and French (1992) three factors are collected from Ken French’s website <a href="http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html">http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html</a>.</td>
</tr>
<tr>
<td><strong>DARKSHARE</strong></td>
<td>Dark pool market share is calculated as share volume executed in alternative trading systems (ATSs) divided by total share trading volume. Weekly share volume executed in ATSs is collected from FINRA OTC Transparency database <a href="https://otctransparency.finra.org/otctransparency/AtsIssueData">https://otctransparency.finra.org/otctransparency/AtsIssueData</a> and total trading volume is collected from CRSP.</td>
</tr>
<tr>
<td><strong>HIS_DARKSHARE</strong></td>
<td>Historical dark pool market share is volume-weighted average of weekly market-adjusted DARKSHARE over one-year window ending 8 weeks before the event week. Market-adjusted DARKSHARE is DARKSHARE of the firm minus market-cap-weighted average of DARKSHARE for all common stocks listed in U.S. Event week is the week during which the Schedule 13D filer’s ownership exceeds the 5% threshold. A minimum of 26 weeks of data is required.</td>
</tr>
<tr>
<td><strong>INFOR</strong></td>
<td>The value of private information possessed by the Schedule 13D filer is defined as (\text{CAR}(-46, +11)).</td>
</tr>
<tr>
<td><strong>JUMP</strong></td>
<td>The ratio of (\text{CAR}(-1, +11)) to (\text{CAR}(-46, +11)). Following Weller (2017), JUMP is defined only when (\left</td>
</tr>
<tr>
<td><strong>TARGET</strong></td>
<td>Indicator variable equal to one if the firm is a target of Schedule 13D filing and zero otherwise.</td>
</tr>
<tr>
<td><strong>TO</strong></td>
<td>Weekly share turnover is calculated as total share trading volume divided by number of shares outstanding (in hundreds).</td>
</tr>
<tr>
<td><strong>TO_DARK</strong></td>
<td>Weekly share turnover in dark pools is calculated as share volume executed in alternative trading systems (ATSs) divided by number of shares outstanding (in hundreds).</td>
</tr>
<tr>
<td><strong>TO_EXCH</strong></td>
<td>Weekly share turnover on exchanges is calculated as share volume executed on exchanges divided by number of shares outstanding (in hundreds). Share volume executed on exchanges (i.e., exchange code ≠ ‘D’) is collected from daily TAQ (DTAQ).</td>
</tr>
<tr>
<td><strong>W13D</strong></td>
<td>Indicator variable equal to one for weeks with disclosed Schedule 13D trades and zero otherwise.</td>
</tr>
</tbody>
</table>
### Control Variables: Firm-week level analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRADESIZE</td>
<td>Average trade size is calculated as weekly total dollar trading volume divided by weekly total number of trades in DTAQ.</td>
</tr>
<tr>
<td>QSPREAD</td>
<td>Share-volume-weighted-average percentage quoted spread computed over all trades during the week. Percentage quoted spread is calculated as the difference between the log national best offer price and log national best bid price one nanosecond before each trade [see Holden et al. (2014) for details].</td>
</tr>
<tr>
<td>DEP</td>
<td>Share-volume-weighted-average dollar depth at national best bid and offer prices (NBBO) computed over all trades during the week. Dollar depth is the average depth at the bid and ask prices one nanosecond before each trade.</td>
</tr>
<tr>
<td>ABSOIMB</td>
<td>Absolute value of order imbalance during the week. Order imbalance is defined as buyer-initiated share volume minus seller-initiated share volume, divided by total share volume. We use the Lee and Ready (1991) algorithm to determine whether a trade is buyer- or seller-initiated.</td>
</tr>
<tr>
<td>RETVAR</td>
<td>Intraday return variance over the week is defined as share-volume-weighted-average daily intraday variance. Daily intraday variance is $10^6$ times variance of 1-minute mid-quote returns.</td>
</tr>
<tr>
<td>PRICE</td>
<td>Share price (i.e., CRSP PRC) at the end of the latest June.</td>
</tr>
<tr>
<td>MV</td>
<td>Market value of common equity (i.e., CRSP PRC*SHROUT) at the end of the latest June.</td>
</tr>
<tr>
<td>BTM</td>
<td>Book-to-market ratio is defined as book value of common shareholders’ equity (COMPUSTAT CEQ) divided by the market value (CRSP PRC*SHROUT) at the end of the latest June.</td>
</tr>
<tr>
<td>BETA</td>
<td>Equity beta estimated from the market model using daily returns over the one-year window ending in the previous month. A minimum of 120 trading days is required.</td>
</tr>
<tr>
<td>VOLATILITY</td>
<td>Idiosyncratic volatility is the standard deviation of residual returns, estimated from the market model, over the one-year window ending in the previous month. A minimum of 120 trading days is required.</td>
</tr>
<tr>
<td>RET1</td>
<td>Market-adjusted return of previous month.</td>
</tr>
<tr>
<td>RET11</td>
<td>Market-adjusted return of the eleven-month window ending before the previous month.</td>
</tr>
</tbody>
</table>

### Control variables: Filing level analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRICE</td>
<td>Share price at the end of eighth week before the event week.</td>
</tr>
<tr>
<td>MV</td>
<td>Market value of common equity at the end of eighth week before the event week.</td>
</tr>
<tr>
<td>BTM</td>
<td>Book-to-market ratio at the end of eighth week before the event week.</td>
</tr>
<tr>
<td>BETA</td>
<td>Equity beta measured at the end of eighth week before the event week.</td>
</tr>
<tr>
<td>VOLATILITY</td>
<td>Idiosyncratic volatility measured at the end of eighth week before the event week.</td>
</tr>
<tr>
<td>RET1</td>
<td>One-month market-adjusted return ending in the eighth week before the event week.</td>
</tr>
<tr>
<td>RET11</td>
<td>Eleven-month market-adjusted return ending in the twelfth week before the event week.</td>
</tr>
</tbody>
</table>

### Transformation of Variables

| LN(X) | Natural logarithm of variable X.                                                                                                          |
Figure 1. Time Line of the Two-period Model

\[ \begin{array}{|c|c|c|}
\hline
0 & 1 & 2 \\
\hline
\tilde{v}_g \text{ is realized.} & \tilde{u}_e, \tilde{u}_d^b, \tilde{u}_d^s \text{ are realized.} & \text{Exchange: market marker sets } \tilde{p} \text{ after seeing } y = \tilde{u}_e + \tilde{x}_e. \\
\text{The informed trader chooses } x_e \text{ and } x_d. & & \text{The dark pool matches orders using } \tilde{p}. \\
\hline
\end{array} \]
Figure 2. The Cumulative Distribution Function of the Power-Law Distribution

This figure illustrates the cumulative distribution function (CDF) of the power-law distribution for different $k$. The x-axis is the order size and the y-axis is the value of the CDF. A distribution with larger $k$ stochastically dominates a distribution with smaller $k$. 
Figure 3. Price Discovery

This figure demonstrates the information revealed through trade. $\sigma_e$ is the proxy for liquidity trading in the exchange; $\sigma_v$ is the fundamental value uncertainty; and $k$ is the proxy for liquidity trading in the dark pool. For all three panels, $\sigma_e = 100$, $\sigma_v = 1$, $k = 1$, and $c = 0.01$. 
Figure 4. Trading Strategy of Schedule 13D Filers

The solid line (right axis) plots the probability that a Schedule 13D filer trades at least one share in a given week from the seventh week before the event week (i.e., the week of crossing 5% ownership threshold) to the week before the filing week (i.e., the week of filing the Schedule 13D with the SEC). For each week, the probability that a Schedule 13D filer trades at least one share is the number of filings with a trade by the filer divided by the total number of Schedule 13D filings in the sample. The solid bars (left axis) represent the percentage of outstanding shares bought by Schedule 13D filers. For each Schedule 13D filing and each week, we calculate the percentage of outstanding shares bought by the filer as the ratio between the number of shares bought by the filer and the number of share outstanding. If no trade is reported on a given week by the filer, the percentage of outstanding shares bought by the filer is set to zero. Then, for each week, the percentage of outstanding shares bought by Schedule 13D filers is the average (across all filings) of the percentage of outstanding shares bought.
Figure 5. Cumulative Abnormal Return around the Filing of Schedule 13Ds

The solid line plots the average cumulative abnormal return (CAR), relative to the Fama and French (1992) three-factor model, from the seventh week before the event week to the seventh week after the filing week of Schedule 13Ds.
Figure 6. Abnormal Market Share of Dark Pools around the Filing of Schedule 13Ds

The solid line plots the average abnormal share turnover executed in dark pools, average abnormal share turnover executed on exchanges, and average abnormal market share of dark pools from the seventh week before the event week to the seventh week after the filing week of Schedule 13Ds. The square dot and dash lines plot the lower and upper bounds of 95% confidence internals of the mean respectively. Abnormal share turnover (market share) is defined as the share turnover (market share) of target firms in excess of value-weighted share turnover (market share) of the market portfolio.
### Table 1
Descriptive Statistics

#### Panel A: Summary statistics for Schedule 13D filings

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>MEAN</th>
<th>MEDIAN</th>
<th>STD</th>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock ownership increase</td>
<td>418</td>
<td>4.78%</td>
<td>3.94%</td>
<td>4.95%</td>
<td>1.85%</td>
<td>6.21%</td>
</tr>
<tr>
<td>CAR (-1,+1)</td>
<td>418</td>
<td>2.69%</td>
<td>1.76%</td>
<td>6.10%</td>
<td>-0.65%</td>
<td>5.56%</td>
</tr>
<tr>
<td>CAR (-1,+11)</td>
<td>418</td>
<td>4.05%</td>
<td>2.90%</td>
<td>10.11%</td>
<td>-2.00%</td>
<td>9.47%</td>
</tr>
<tr>
<td>CAR (-46,+11)</td>
<td>418</td>
<td>9.62%</td>
<td>8.30%</td>
<td>30.03%</td>
<td>-6.00%</td>
<td>24.40%</td>
</tr>
<tr>
<td>Jump Ratio of CAR (-1,+11) to CAR (-46,+11)</td>
<td>182</td>
<td>20.13%</td>
<td>17.42%</td>
<td>35.27%</td>
<td>1.83%</td>
<td>39.14%</td>
</tr>
<tr>
<td>Historical dark pool market share</td>
<td>325</td>
<td>0.87%</td>
<td>1.20%</td>
<td>4.61%</td>
<td>-1.68%</td>
<td>4.09%</td>
</tr>
</tbody>
</table>

#### Panel B: Summary statistics for the matched sample of firm-weeks

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>MEAN</th>
<th>MEDIAN</th>
<th>STD</th>
<th>Q1</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO</td>
<td>6,416</td>
<td>5.16</td>
<td>3.16</td>
<td>6.04</td>
<td>1.47</td>
<td>6.25</td>
</tr>
<tr>
<td>TO_DARK</td>
<td>6,416</td>
<td>0.83</td>
<td>0.48</td>
<td>0.99</td>
<td>0.18</td>
<td>1.07</td>
</tr>
<tr>
<td>TO_EXCH</td>
<td>6,415</td>
<td>3.17</td>
<td>2.01</td>
<td>3.60</td>
<td>0.87</td>
<td>3.89</td>
</tr>
<tr>
<td>DARKSHARE</td>
<td>6,416</td>
<td>15.34%</td>
<td>15.05%</td>
<td>7.55%</td>
<td>10.07%</td>
<td>19.85%</td>
</tr>
<tr>
<td>WI3D</td>
<td>6,400</td>
<td>58.70%</td>
<td>100.00%</td>
<td>49.24%</td>
<td>0.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>BUY13D ($)</td>
<td>1,882</td>
<td>9,758,772</td>
<td>1,229,588</td>
<td>26,743,838</td>
<td>229,499</td>
<td>6,487,460</td>
</tr>
<tr>
<td>BUY13D (%)</td>
<td>1,892</td>
<td>0.84</td>
<td>0.38</td>
<td>1.34</td>
<td>0.11</td>
<td>0.95</td>
</tr>
<tr>
<td>TRADESCIZE</td>
<td>6,416</td>
<td>3,120</td>
<td>1,953</td>
<td>3,312</td>
<td>1,075</td>
<td>3,901</td>
</tr>
<tr>
<td>QSPREAD</td>
<td>6,416</td>
<td>1.03%</td>
<td>0.49%</td>
<td>1.60%</td>
<td>0.22%</td>
<td>1.16%</td>
</tr>
<tr>
<td>DEP</td>
<td>6,416</td>
<td>28,120</td>
<td>10,036</td>
<td>59,649</td>
<td>4,876</td>
<td>24,102</td>
</tr>
<tr>
<td>ABSOIMB</td>
<td>6,416</td>
<td>12.81%</td>
<td>8.21%</td>
<td>13.85%</td>
<td>3.71%</td>
<td>16.55%</td>
</tr>
<tr>
<td>RETVAR</td>
<td>6,416</td>
<td>16.28%</td>
<td>3.07%</td>
<td>57.90%</td>
<td>1.12%</td>
<td>7.50%</td>
</tr>
<tr>
<td>PRICE</td>
<td>6,416</td>
<td>24.55</td>
<td>13.88</td>
<td>32.57</td>
<td>6.52</td>
<td>29.26</td>
</tr>
<tr>
<td>MV</td>
<td>6,416</td>
<td>1,572,438</td>
<td>431,106</td>
<td>5,131,170</td>
<td>146,779</td>
<td>1,286,054</td>
</tr>
<tr>
<td>BTM</td>
<td>6,416</td>
<td>0.60</td>
<td>0.52</td>
<td>0.58</td>
<td>0.27</td>
<td>0.85</td>
</tr>
<tr>
<td>BETA</td>
<td>6,416</td>
<td>1.04</td>
<td>1.03</td>
<td>0.54</td>
<td>0.69</td>
<td>1.35</td>
</tr>
<tr>
<td>VOLATILITY</td>
<td>6,416</td>
<td>2.66%</td>
<td>2.35%</td>
<td>1.45%</td>
<td>1.68%</td>
<td>3.20%</td>
</tr>
<tr>
<td>RET1</td>
<td>6,416</td>
<td>-0.77%</td>
<td>-1.28%</td>
<td>12.38%</td>
<td>-7.02%</td>
<td>4.70%</td>
</tr>
<tr>
<td>RET11</td>
<td>6,416</td>
<td>-11.23%</td>
<td>-14.93%</td>
<td>39.61%</td>
<td>-35.76%</td>
<td>6.29%</td>
</tr>
</tbody>
</table>

#### Panel C: Comparison between weeks with and without Schedule 13D trades

<table>
<thead>
<tr>
<th></th>
<th>(1) Weeks with Schedule 13D trades (Obs. = 1,892)</th>
<th>(2) Weeks without Schedule 13D trades (Obs. = 1,339)</th>
<th>(3) Same weeks of matched firms (Obs. = 1,881)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Mean</td>
</tr>
<tr>
<td>TO</td>
<td>7.12</td>
<td>4.58</td>
<td>4.46***</td>
</tr>
<tr>
<td>TO_DARK</td>
<td>1.19</td>
<td>0.75</td>
<td>0.70***</td>
</tr>
</tbody>
</table>
Panel A presents descriptive statistics of Schedule 13D filings. Stock ownership increase is the net increase in total beneficial ownership of the Schedule 13D filer over the 60 calendar days prior to the filing date, calculated as the net number of purchased shares disclosed in Schedule 13D filings divided by number of shares outstanding. Cumulative abnormal return (CAR), relative to the Fama and French (1992) three-factor model, is calculated over different windows, whose starting and ending dates are specified relative to the filing date (i.e., the day on which Schedule 13D filing is submitted to the SEC). Jump ratio is defined as $\text{CAR}(-1,+11)$ divided by $\text{CAR}(-46,+11)$ for sufficiently large $\text{CAR}(-46,+11)$ following Weller (2017). Historical market-adjusted dark pool market share is the average weekly market-adjusted dark pool market share of the target firm over the one-year window ending 8 weeks before the event week, the week during which the 13D filer’s ownership exceeds the 5% threshold. Market-wide dark pool share is the value-weighted average across all common stock listed in U.S. Panel B presents descriptive statistics for the matched sample that consists of the weeks in the disclosure period for both the target firms and matched firms. The disclosure period for Schedule 13D trades covers the seven weeks before the event week, the event week, and the week before the filing week if any. Matched stocks are created using the Propensity-Score method (see Section 5.3 for further details). Panel C presents descriptive statistics for subsets of the matched sample. Column (1) reports the mean and median of variables for weeks with Schedule 13D trades. Similarly, column (2) reports the mean and median of variables for weeks without Schedule 13D trades of target firms. Column (3) replicates column (1) for the same calendar weeks of matched firms. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed), respectively, when comparing columns (2) or (3) with column (1).
<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase of Dark Pool Market Share during Weeks with Schedule 13D Trades</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$W_{13D}$</td>
</tr>
<tr>
<td>$W_{13D}\cdot\text{TARGET}$</td>
</tr>
<tr>
<td>$\ln(\text{BUY}_{13D})$</td>
</tr>
<tr>
<td>$\text{BUY}_{13D}\cdot\text{RATIO}$</td>
</tr>
<tr>
<td>$\ln(\text{TRADESIZE})$</td>
</tr>
<tr>
<td>$\ln(\text{DEP})$</td>
</tr>
<tr>
<td>$\text{ABSOIMB}$</td>
</tr>
<tr>
<td>$\ln(\text{RETVAR})$</td>
</tr>
<tr>
<td>$\ln(\text{PRICE})$</td>
</tr>
<tr>
<td>$\ln(\text{MV})$</td>
</tr>
<tr>
<td>$\ln(\text{VOLATILITY})$</td>
</tr>
</tbody>
</table>

Event Fixed Effects: Yes, Yes, Yes

Obs.: 6,440, 6,440, 6,440

$R^2$: 47%, 47%, 47%

This table shows the relation between Schedule 13D trades and dark pool market share. We estimate the following difference-in-differences regression on the matched sample that consists of weeks during the disclosure period of both target and matched non-target firms: $DARKSHARE_{i,t} = \beta_1 \cdot W_{13D_{i,t}} + \beta_2 \cdot W_{13D_{i,t}} \cdot \text{TARGET}_i + \beta_3 \cdot \ln(\text{BUY}_{13D_{i,t}}) + \beta_4 \cdot \text{BUY}_{13D}\cdot\text{RATIO}_{i,t} + \sum \text{CONTROL}_{i,t} + \gamma_i + \varepsilon_{i,t}$. The disclosure period for Schedule 13D trades covers the seven weeks before the event week, the event week, and the week before the filing week if any. Matched stocks are created using the Propensity-Score method (see Section 5.3 for further details). $\text{TARGET}$ is an indicator variable equal to one for firms targeted by Schedule 13D filings and zero for the propensity-score matched non-target firms. $W_{13D}$ is an indicator variable equal to one for weeks with trades by Schedule 13D filers and zero for weeks without 13D trades. $W_{13D}$ for matched firms is defined to be the same as corresponding target firms. $\gamma_i$ are event fixed effects. $\text{BUY}_{13D}$ is the dollar value of net shares acquired by the Schedule 13D filer during a week. $\text{BUY}_{13D}\cdot\text{RATIO}$ is the ratio of net shares acquired by the Schedule 13D filer divided by total shares traded during a week. $\ln(\text{BUY}_{13D})$ or $\text{BUY}_{13D}\cdot\text{RATIO}$ is set to zero for non-target matched stocks. $\sum \text{CONTROL}_{i,t}$ are control variables. In each column, we report estimated coefficients and their t-statistics calculated using standard errors clustered by event. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed), respectively.
Table 3
Cross-sectional Variation in the Increase of Dark Pool Market Share during Weeks with Schedule 13D Trades

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W13D)</td>
<td>0.006***</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>2.94</td>
<td>-0.28</td>
<td>-0.29</td>
</tr>
<tr>
<td>(W13D*\text{INFOR})</td>
<td>0.024**</td>
<td></td>
<td>0.028**</td>
</tr>
<tr>
<td></td>
<td>2.16</td>
<td></td>
<td>2.36</td>
</tr>
<tr>
<td>(W13D*\text{HIS_DARKSHARE})</td>
<td>0.002***</td>
<td>0.115***</td>
<td>0.104***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.51</td>
<td>3.99</td>
</tr>
<tr>
<td>(TO)</td>
<td>0.000***</td>
<td>0.002***</td>
<td>0.001***</td>
</tr>
<tr>
<td></td>
<td>7.07</td>
<td>6.02</td>
<td>5.86</td>
</tr>
<tr>
<td>(\text{LN}()_{\text{TRADESIZE}})</td>
<td>-0.009**</td>
<td>-0.007</td>
<td>-0.008*</td>
</tr>
<tr>
<td></td>
<td>-2.11</td>
<td>-1.45</td>
<td>-1.73</td>
</tr>
<tr>
<td>(\text{QSPREAD})</td>
<td>-0.309***</td>
<td>-0.395***</td>
<td>-0.389***</td>
</tr>
<tr>
<td></td>
<td>-2.99</td>
<td>-3.65</td>
<td>-3.58</td>
</tr>
<tr>
<td>(\text{LN}()_{\text{DEP}})</td>
<td>-0.010***</td>
<td>-0.010***</td>
<td>-0.010***</td>
</tr>
<tr>
<td></td>
<td>-4.99</td>
<td>-4.74</td>
<td>-4.83</td>
</tr>
<tr>
<td>(\text{ABSOIMB})</td>
<td>-0.055***</td>
<td>-0.057***</td>
<td>-0.057***</td>
</tr>
<tr>
<td></td>
<td>-6.62</td>
<td>-6.35</td>
<td>-6.29</td>
</tr>
<tr>
<td>(\text{LN}()_{\text{RETVAR}})</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>0.24</td>
<td>0.32</td>
</tr>
<tr>
<td>(\text{LN}()_{\text{PRICE}})</td>
<td>0.009</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>0.71</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>(\text{LN}()_{\text{MV}})</td>
<td>-0.016</td>
<td>-0.016</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>-1.14</td>
<td>-1.05</td>
<td>-1.03</td>
</tr>
<tr>
<td>(\text{LN}()_{\text{VOLATILITY}})</td>
<td>0.032**</td>
<td>0.030*</td>
<td>0.029*</td>
</tr>
<tr>
<td></td>
<td>2.21</td>
<td>1.94</td>
<td>1.87</td>
</tr>
<tr>
<td>Event Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>6,440</td>
<td>5,751</td>
<td>5,751</td>
</tr>
<tr>
<td>(R^2)</td>
<td>47%</td>
<td>48%</td>
<td>48%</td>
</tr>
</tbody>
</table>

This table shows cross-sectional variation in the relation between Schedule 13D trades and dark pool market share. We estimate the following difference-in-differences regression on the matched sample that consists of weeks in the disclosure period for both target and matched non-target firms:

\[
\text{DARKSHARE}_{i,t} = \beta_1 \cdot W13D_{i,t} + \beta_2 \cdot W13D_{i,t} \cdot \text{INFOR}_i + \beta_3 \cdot W13D_{i,t} \cdot \text{HIS\_DARKSHARE}_i + \sum \text{CONTROL}_{i,t} + \gamma_t + \epsilon_{i,t}.
\]

\(W13D\) is an indicator equal to one for weeks with Schedule 13D trades and zero for weeks without 13D trades. \(W13D\) for matched firms is defined to be the same as corresponding target firms. \(\text{INFOR}\) is cumulative abnormal return of the target firm over the \((t - 46, 11)\) trading-day window around the filing date. \(\text{HIS\_DARKSHARE}\) is the average weekly market-adjusted dark pool market share of the target firm over the one-year window ending 8 weeks before the event week. \(\text{INFOR}\) and \(\text{HIS\_DARKSHARE}\) are set to zero for non-target matched stocks. \(\gamma_t\) are event fixed effects. \(\sum \text{CONTROL}_{i,t}\) are control variables. In each column, we report estimated coefficients and their \(t\)-statistics calculated using standard errors clustered by event. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed), respectively.
### Table 4
Cross-sectional Variation in the Price Discovery of Schedule 13D Events

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INFOR</strong></td>
<td>0.119*</td>
<td>0.180**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>HIS_DARKSHARE</strong></td>
<td>1.724*</td>
<td>1.788*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>LN(PRICE)</strong></td>
<td>-0.064</td>
<td>-0.095*</td>
<td>-0.093*</td>
</tr>
<tr>
<td></td>
<td>-1.41</td>
<td>-1.78</td>
<td>-1.77</td>
</tr>
<tr>
<td><strong>LN(MV)</strong></td>
<td>0.004</td>
<td>-0.005</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>-0.13</td>
<td>-0.21</td>
</tr>
<tr>
<td><strong>BTM</strong></td>
<td>0.031</td>
<td>0.077*</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>0.77</td>
<td>1.65</td>
<td>1.44</td>
</tr>
<tr>
<td><strong>BETA</strong></td>
<td>0.021</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.15</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>LN(VOLATILITY)</strong></td>
<td>-0.216**</td>
<td>-0.255**</td>
<td>-0.265**</td>
</tr>
<tr>
<td></td>
<td>-2.34</td>
<td>-2.44</td>
<td>-2.57</td>
</tr>
<tr>
<td><strong>RET1</strong></td>
<td>0.428*</td>
<td>0.685**</td>
<td>0.620**</td>
</tr>
<tr>
<td></td>
<td>1.69</td>
<td>2.38</td>
<td>2.17</td>
</tr>
<tr>
<td><strong>RET11</strong></td>
<td>-0.006</td>
<td>0.015</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td>-0.06</td>
<td>0.14</td>
<td>0.97</td>
</tr>
<tr>
<td><strong>INTERCEPT</strong></td>
<td>-0.57</td>
<td>-0.554</td>
<td>-0.571</td>
</tr>
<tr>
<td></td>
<td>-1.39</td>
<td>-1.22</td>
<td>-1.27</td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>177</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td>8%</td>
<td>10%</td>
<td>14%</td>
</tr>
</tbody>
</table>

This table shows cross-sectional variation in price discovery process of Schedule 13D events. We estimate the following regression on the sample of Schedule 13D filings: \( JUMP_i = \alpha + \beta_1 \cdot INFOR_i + \beta_2 \cdot HIS_DARKSHARE_i + \sum CONTROL_i + \epsilon_i \). \( JUMP \) is defined as \( CAR \) of \((t - 1, t + 1)\) divided by \( CAR \) of \((t - 46, t + 1)\) window around the filing date. Following Weller (2017), we drop events with small values of \( JUMP \) ratio denominator. \( INFOR \) represents the total amount of private information possessed by the Schedule 13D filer, calculated as \( CAR \) of \((t - 46, t + 11)\) trading-day window around the filing date. \( HIS_DARKSHARE \) is the average weekly market-adjusted dark pool market share of the target firm over the one-year window ending 8 weeks before the event week. Market-wide dark pool share is the value-weighted average across all common stock listed in U.S. All other control variables are measured at the end of eighth week before the event week. In each column, we report estimated coefficients and their \( t \)-statistics. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels (two-tailed), respectively.